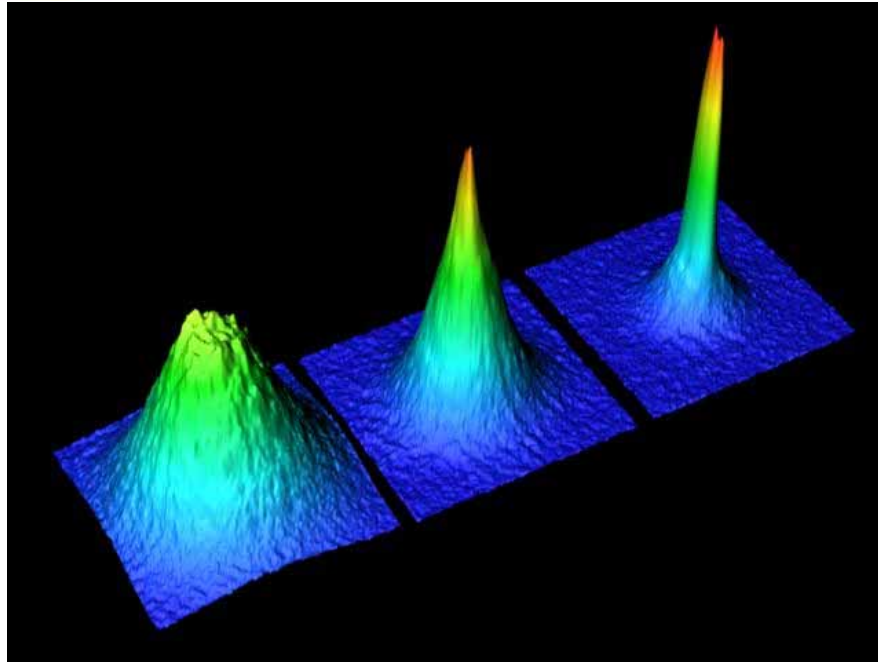


4 . 原子気体のボース・アインシュタイン凝縮 (BEC)



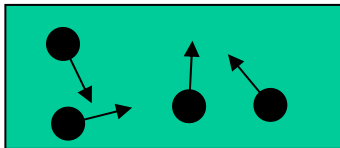
2001 E. Cornell, C. Wieman, W. Ketterle



$$\rho_{PSD} = n\lambda_{dB}^3 = n\left(h / \sqrt{2\pi m_A k_B T}\right)^3$$

位相空間密度: > 2.612

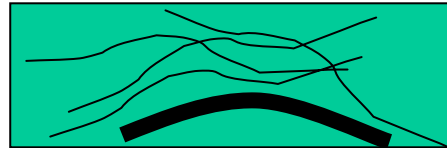
$T_C = 100 \text{ nK}$, $n = 10^{14} / \text{cm}^3$



高温: 原子はランダムに熱運動をしています。

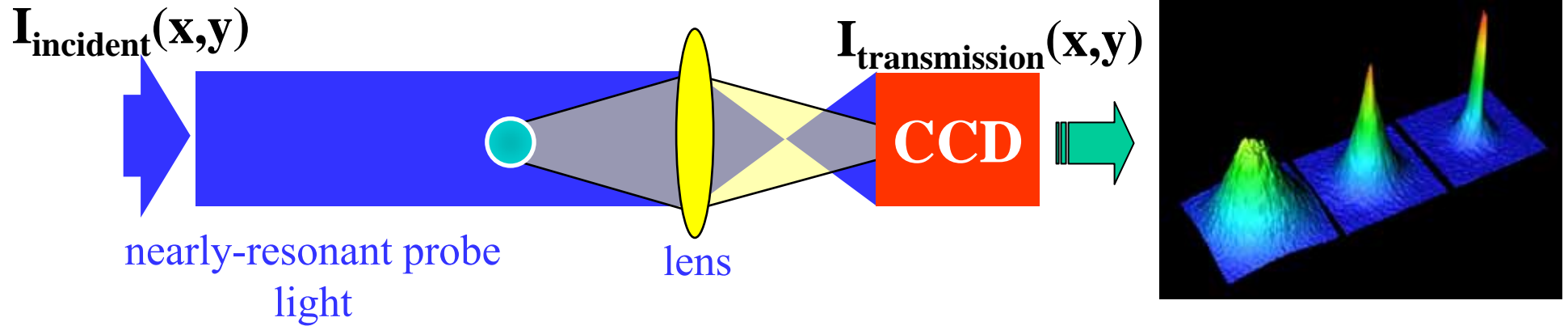


低温: レーザー冷却法により低温になった原子では、波動性が顕著に表れます。



極低温: さらに冷却されるとお互いの波が重なり合い、純粋に量子力学的な相転移が起きます。これがボース・アインシュタイン凝縮(BEC)です。

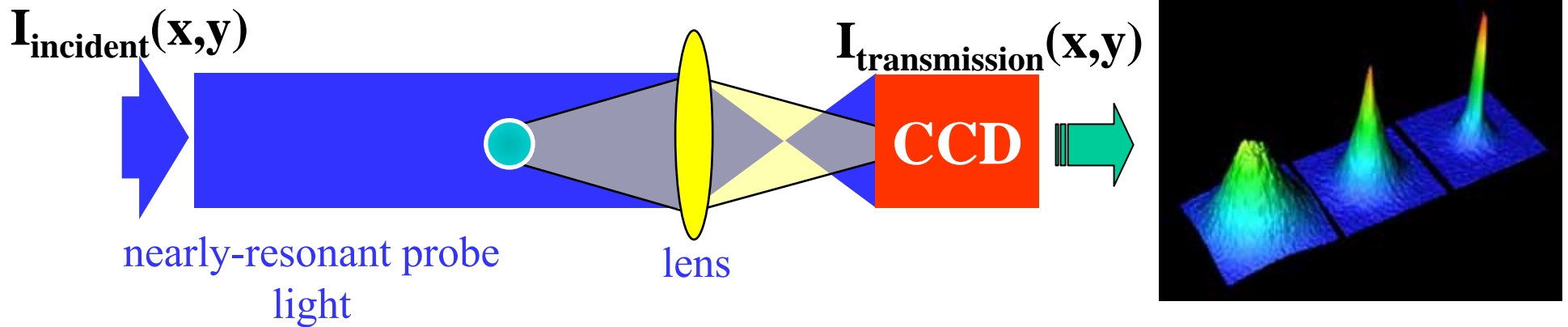
Optical Imaging



$$I_{\text{transmission}}(x, y) = I_{\text{incident}}(x, y) \exp(-\sigma_{\text{abs}} n(x, y) L)$$

$$\longrightarrow n(x, y) = -\frac{1}{\sigma_{\text{abs}} L} \log\left(\frac{I_{\text{transmission}}(x, y)}{I_{\text{incident}}(x, y)}\right)$$

Optical Imaging



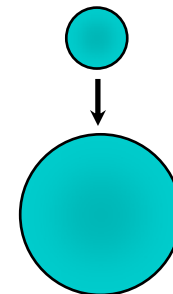
Atom number

$$N = \int_{-\infty}^{+\infty} n(x, y) L dx dy = -\frac{1}{\sigma_{abs}} \int_{-\infty}^{+\infty} \log\left(\frac{I_{transmission}(x, y)}{I_{incident}(x, y)}\right) dx dy$$

Temperature

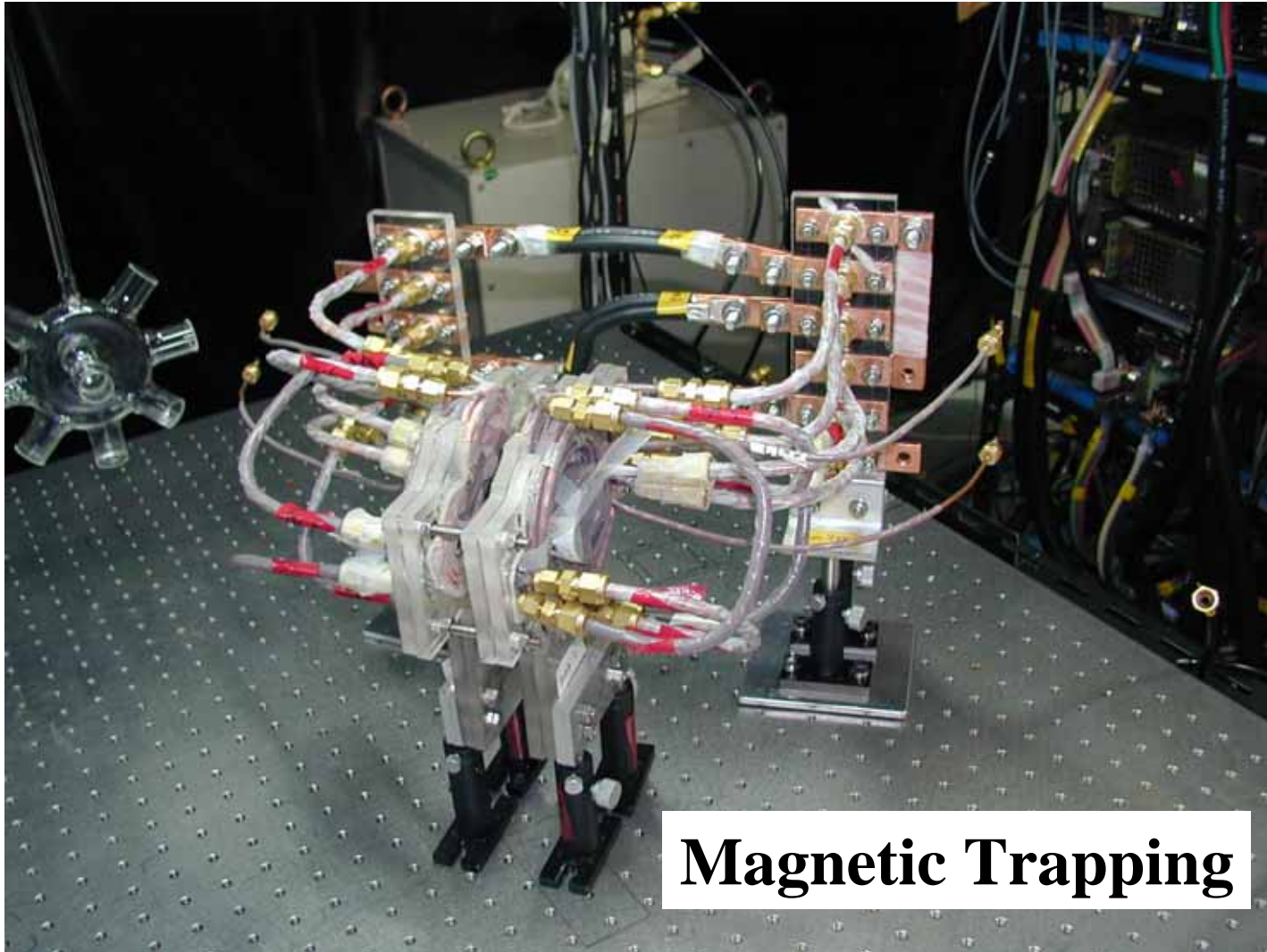
$$T = \frac{M (\sigma_{final}^2 - \sigma_{initial}^2)}{k_B t^2}$$

Time-of-flight measurement



4 - 1 . B E C の生成

(i) 磁気トラップ (Magnetic Trap)



$I \sim 300 \text{ A}$

$B' \sim 200 \text{ G/cm}$

$B'' \sim 160 \text{ G/cm}^2$

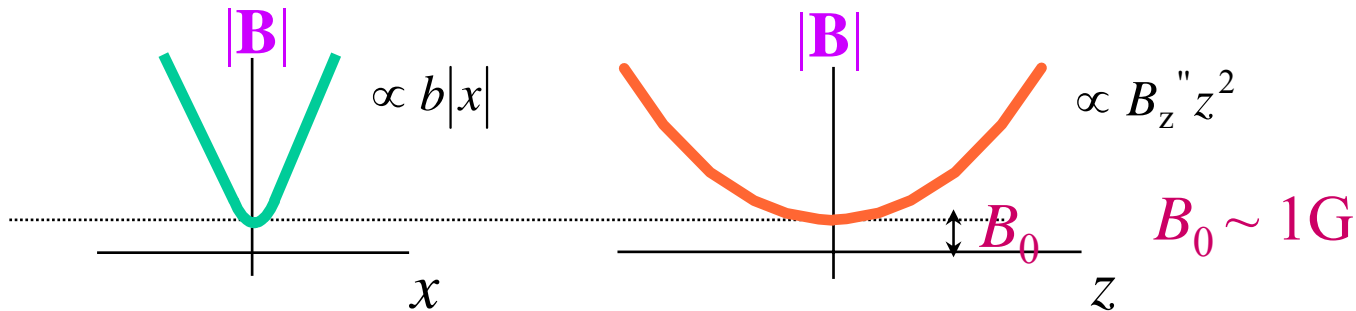
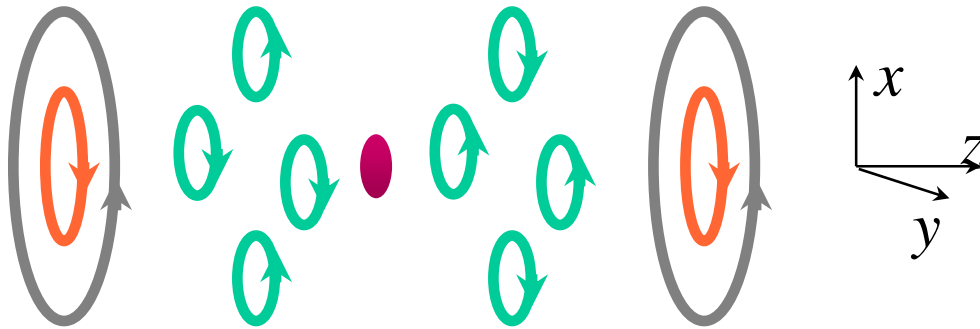
$B_0 \sim 1 \text{ G}$

$r \sim 200 \text{ Hz}$

$z \sim 16 \text{ Hz}$

Ioffe-Pritchard Magnetic Trap

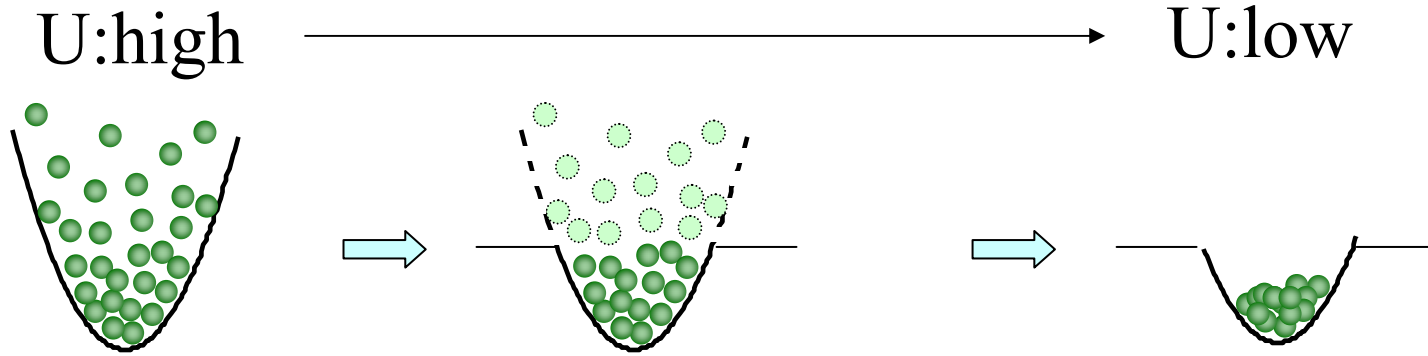
Clover leaf trap (Ioffe-Pritchard trap)



$$\vec{B} = \left(-bx - B''zx, by + B''zy, B_0 + B''\left(z^2 - \frac{x^2 + y^2}{2}\right) \right)$$

$$|\vec{B}| \approx B_0 + B''\left(z^2 - \frac{x^2 + y^2}{2}\right) + \frac{b^2}{2B_0}(x^2 + y^2)$$

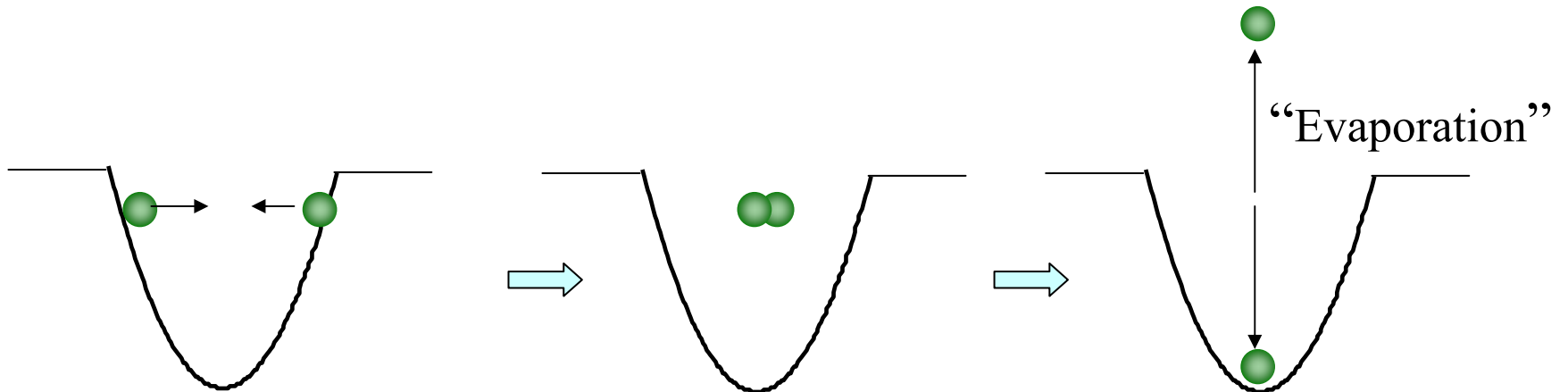
4 - 2 . Evaporative Cooling(蒸発冷却)



N: large
T: high
n: low

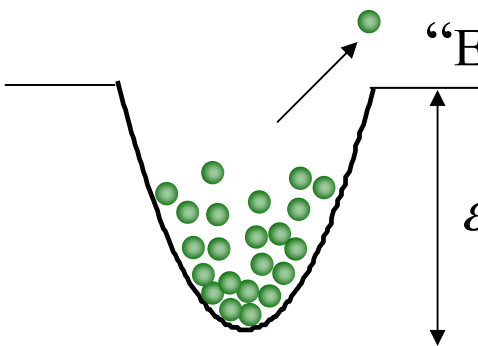
Thermalization
(衝突による
熱平衡化)

N:small
T:low
n:high



Evaporative Cooling(蒸発冷却)

$\bar{\varepsilon}$: average energy of atoms in trap
 ε_{ev} : trap depth
 $\eta \equiv \frac{\varepsilon_{ev}}{k_B T}$: truncation factor
 ε' : average energy of evaporated atoms from trap



N : total atom number in trap
 $E = N\bar{\varepsilon}$: total energy of atoms in trap

Only evaporation considered for atom number loss

$$-dE = -Nd\bar{\varepsilon} - \bar{\varepsilon}dN = \varepsilon' dN = (1 + \beta)dN$$

$$\longrightarrow \frac{d \ln \bar{\varepsilon}}{d \ln N} = -\beta$$

For 3D harmonic trap: $\bar{\varepsilon} = (3/2 + 3/2)k_B T = 3k_B T \longrightarrow \frac{d \ln T}{d \ln N} = -\beta$

Evaporative Cooling(蒸発冷却)

loss other than evaporation also included

$$\frac{dN}{N} \Big|_{loss} = -\frac{dt}{\tau_{loss}}, \quad \frac{dN}{N} \Big|_{ev} = -\frac{dt}{\tau_{ev}} \quad \longrightarrow \quad \frac{dN \Big|_{ev}}{dN \Big|_{ev} + dN \Big|_{loss}} = \frac{\tau_{loss}}{\tau_{loss} + \tau_{ev}}$$

$$\longrightarrow \quad \frac{d \ln T}{d \ln N} = -\beta \frac{\tau_{loss}}{\tau_{loss} + \tau_{ev}} \equiv -\beta'$$

If $\beta \gg 1$, then

$$\frac{dN}{dt} \Big|_{ev} = -N n(0) \sigma \bar{v} \left(\frac{\epsilon_{ev}}{k_B T} \right) \exp\left(-\frac{\epsilon_{ev}}{k_B T}\right) \quad n(0) : \text{peak atom density}$$

$$\bar{v} = \left(\frac{8k_B T}{\pi m} \right)^{1/2} : \text{average atom velocity}$$

$$\longrightarrow \quad \frac{1}{\tau_{ev}} = \frac{1}{\tau_{el}} \left(\frac{\epsilon_{ev}}{\sqrt{2} k_B T} \right) \exp\left(-\frac{\epsilon_{ev}}{k_B T}\right), \quad \frac{1}{\tau_{el}} \equiv n(0) \sqrt{2} \bar{v} \sigma$$

$$\text{If } \epsilon' \approx \epsilon_{ev} \longrightarrow \quad \frac{d \ln T}{d \ln N} = \left(\frac{\epsilon_{ev}}{\bar{\epsilon}} - 1 \right) \left(1 + \frac{\tau_{el}}{\tau_{loss}} \frac{\sqrt{2} k_B T}{\epsilon_{ev}} \exp\left(\frac{\epsilon_{ev}}{k_B T}\right) \right)^{-1}$$



$$\frac{d \ln \rho_{PSD}}{d \ln N} = -3\beta' + 1$$

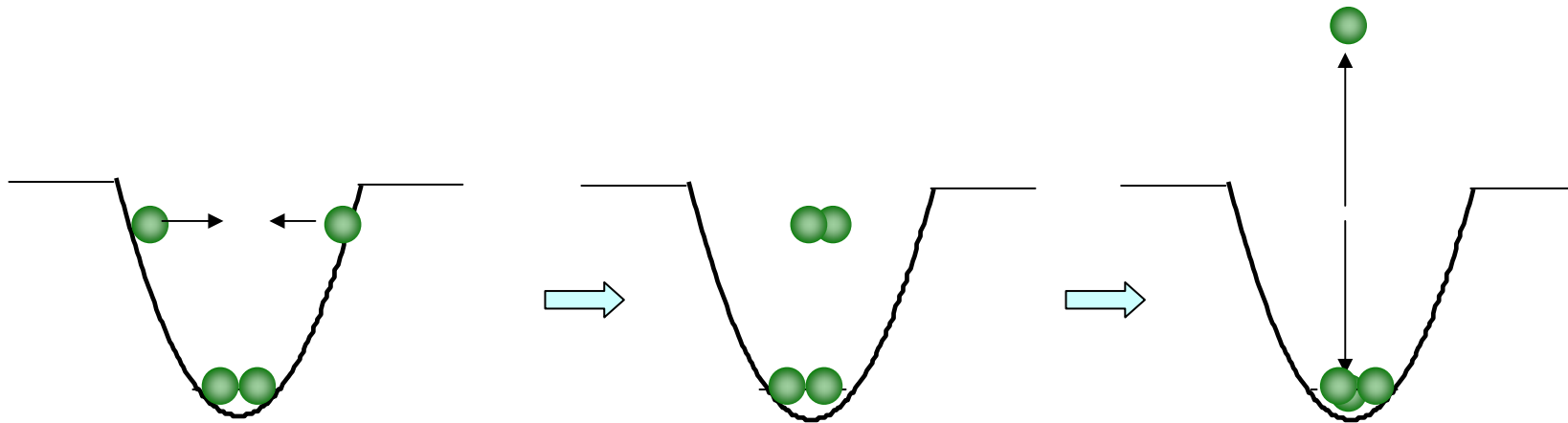
$$\frac{d \ln \tau_{el}}{d \ln N} = \beta' - 1 > 0$$

(runaway evaporation)

Evaporative Cooling(蒸発冷却)

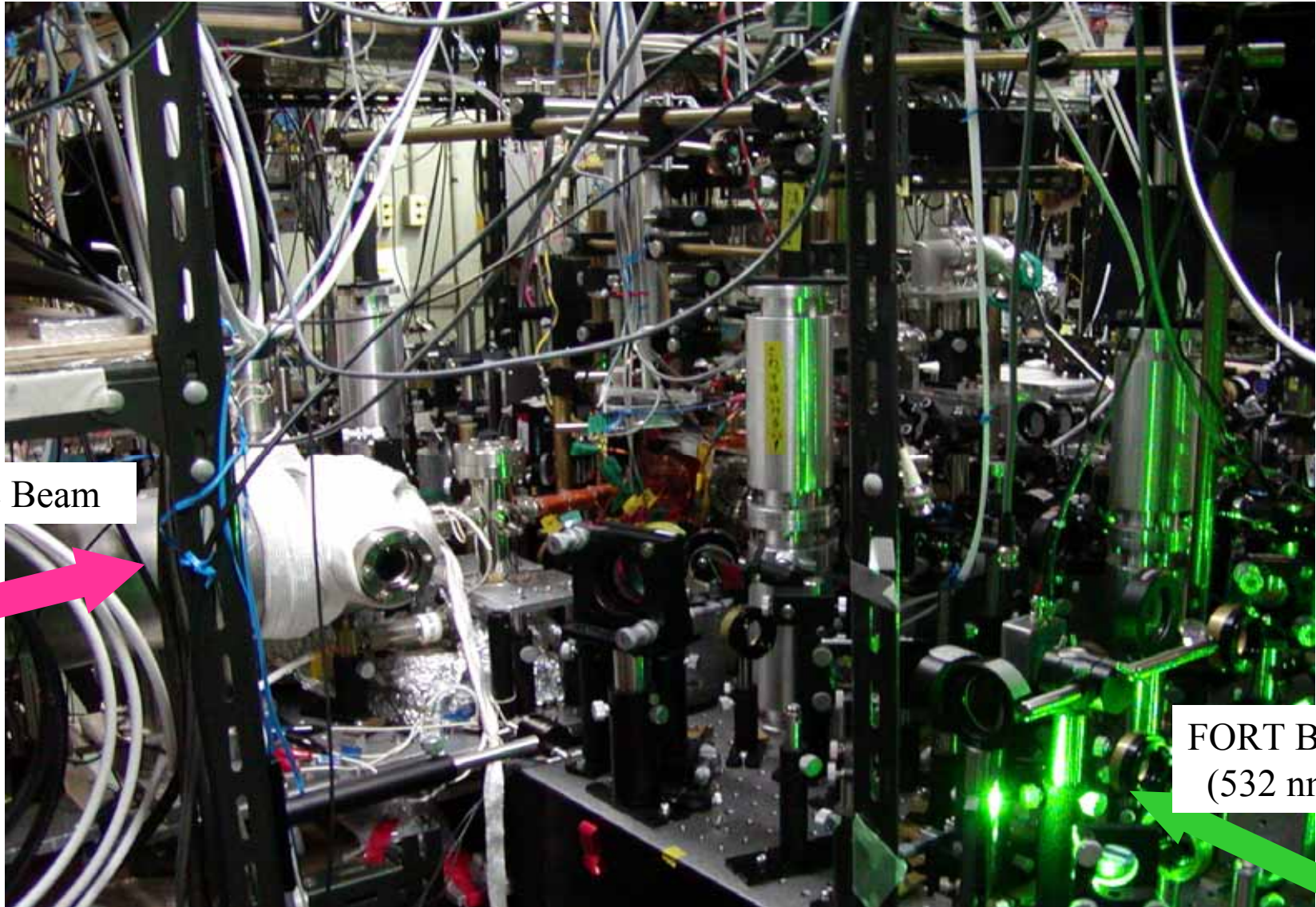
$$T \approx T_C$$

“Evaporation”



Bosonic Stimulation : $R \propto N_{initial} \times (1 + N_{final})$

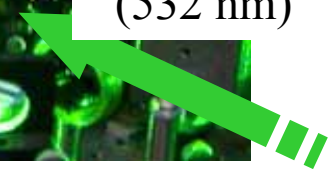
実験装置



Atomic Beam



FORT Beams
(532 nm)



Atomic BEC

1995 ^{87}Rb , ^{23}Na , ^7Li

1998 ^1H

2000 ^{85}Rb

2001 ^{41}K $^4\text{He}^*$

2003 ^{133}Cs ^{174}Yb

2005 ^{52}Cr

4 - 2 . 基本的性質

巨視的な数の原子の波動関数: $\Psi(r_1, \dots, r_N) = \prod_{i=1}^N \phi(r_i)$

single-particle wavefunction $\phi(r)$ Normalization: $\int dr |\phi(r)|^2 = 1$

→ Condensate wavefunction: “order parameter”

$$\Phi(r) \equiv N^{1/2} \phi(r) \quad n(r) = |\Phi(r)|^2, N = \int dr |\Phi(r)|^2$$

ラグランジュの未定乗数法

$$\delta E(\Phi^*, \Phi) - \mu N(\Phi^*, \Phi) = 0 \quad \mu : \text{chemical potential}$$

$$\rightarrow \left(-\frac{\hbar^2}{2m} \Delta + V(r) + U_0 |\Phi(r)|^2 \right) \Phi(r) = \mu \Phi(r) \quad U_0 = \frac{4\pi\hbar^2 a_s}{m}$$

“Gross-Pitaevskii 方程式”

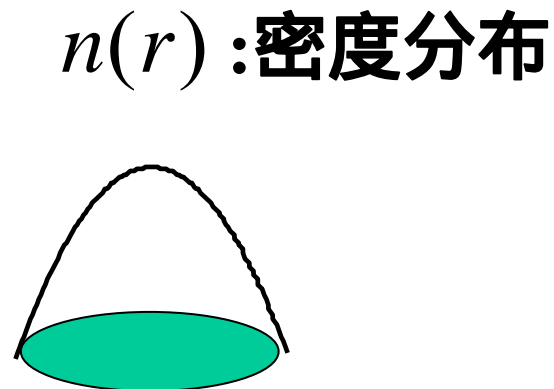
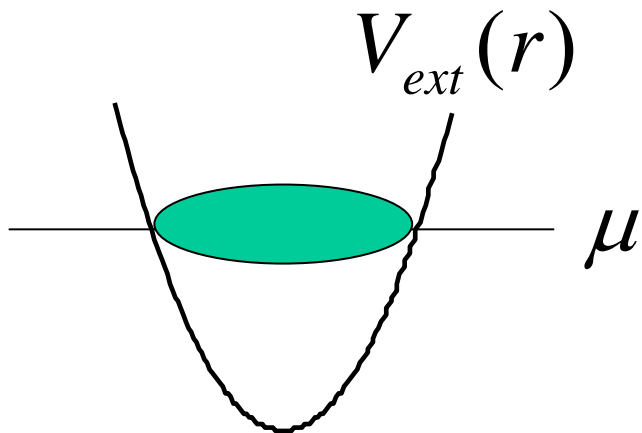
$$a_s : \text{散乱長} \quad \begin{cases} a_s > 0 : \text{repulsive (斥力)} & \longrightarrow \text{安定} \\ a_s < 0 : \text{attractive (引力)} & \longrightarrow \text{不安定: } N < N_c \end{cases}$$

原子气体BEC: Thomas-Fermi近似

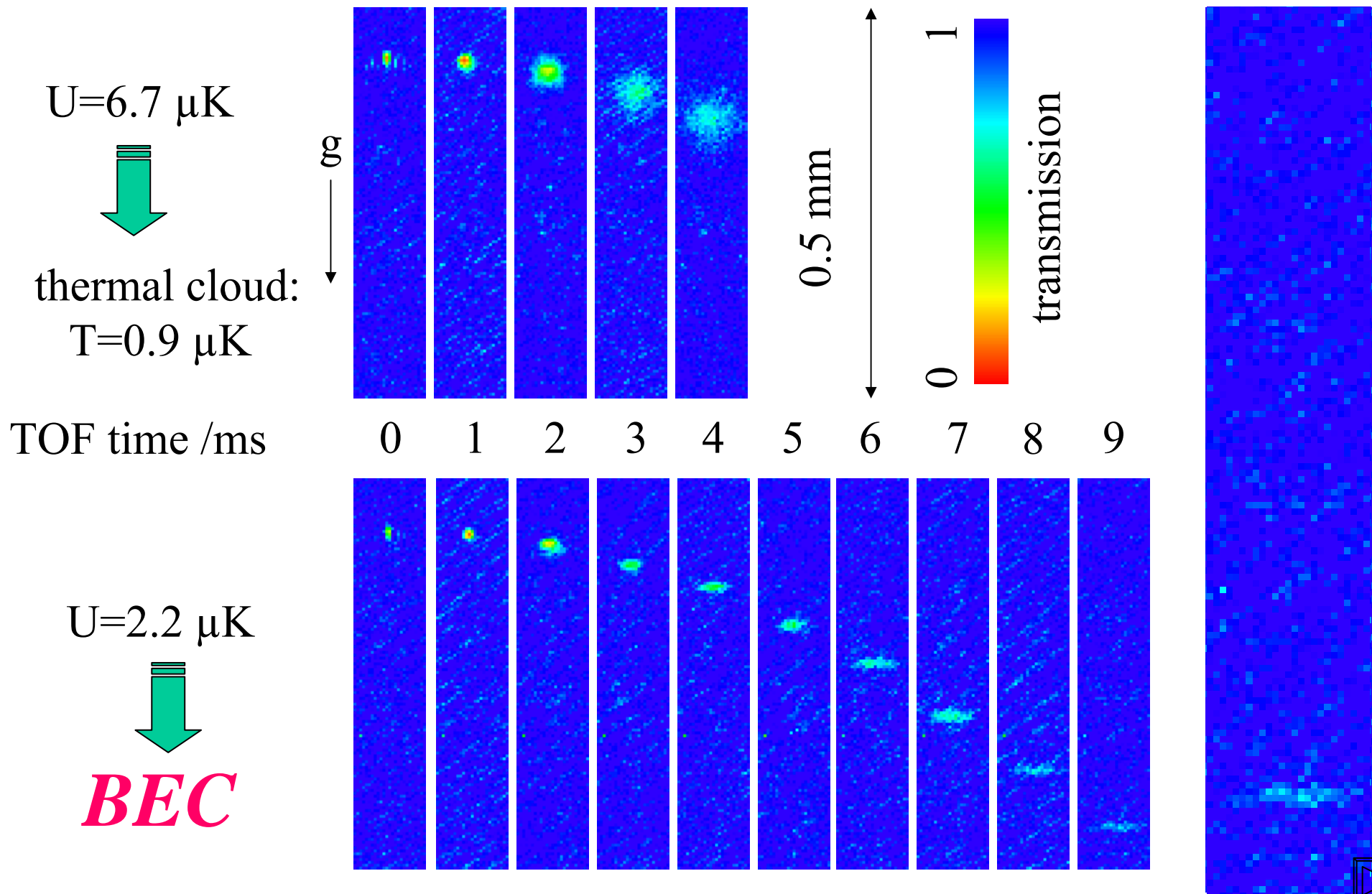
$$\mu\Phi = \left(-\frac{\hbar^2}{2m} \Delta + V_{ext} + U_0 |\Phi|^2 \right) \Phi$$

→
$$\mu\Phi = (V_{ext} + U_0 n(r))\Phi$$

→
$$n(r) = (\mu - V_{ext}) / U_0$$



BEC



非等方的な広がり

BEC in Thomas-Fermi regime: $\omega_x = \omega_y \equiv \omega_r \gg \omega_z$

$$n_{TF}(r, t) = \frac{\mu}{g} \left[1 - \left(\frac{r^2}{d_r^2(t)} + \frac{z^2}{d_z^2(t)} \right) \right] \quad d_{r,z}(0) \equiv \sqrt{\frac{2\mu}{m\omega_{r,z}^2}}$$

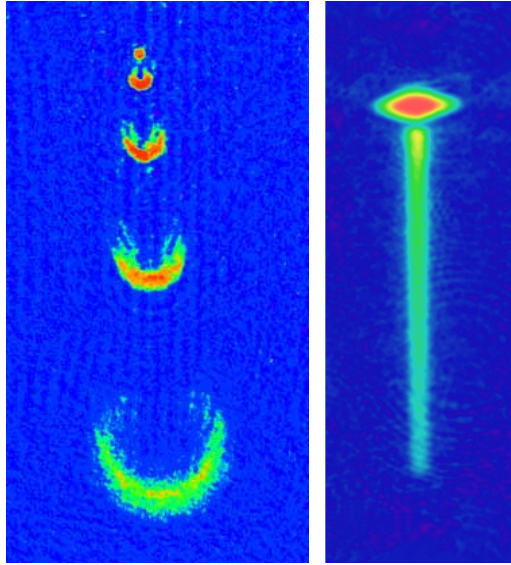
$$\varepsilon_{TF}(t) \equiv \frac{d_r(t)}{d_z(t)} \quad \text{: アスペクト比}$$

$$\longrightarrow \varepsilon_{TF}(0) = \frac{\omega_z}{\omega_r} \ll 1 \quad \varepsilon_{TF}(t) \xrightarrow{t \rightarrow \infty} \frac{2}{\pi} \frac{\omega_r}{\omega_z} \gg 1$$

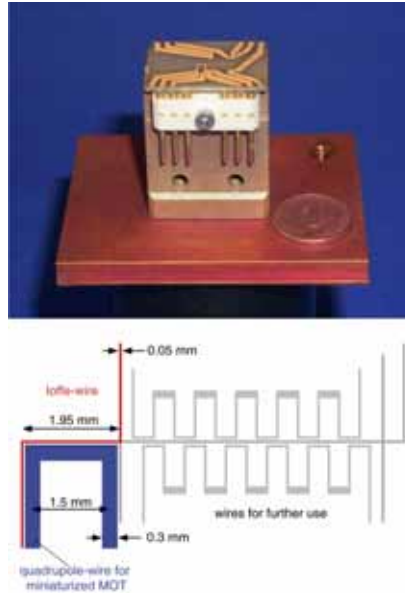


4 - 3 . 様々な応用

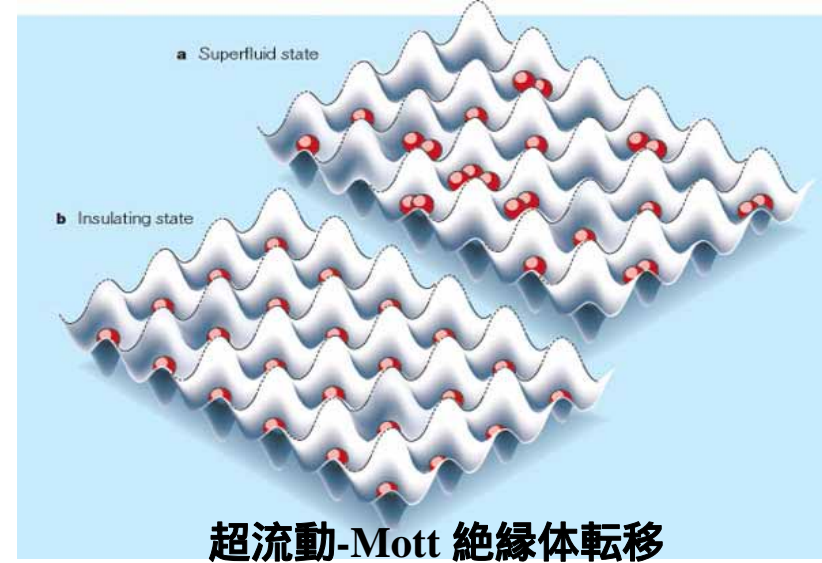
Atom Laser:
コヒーレントな物質波



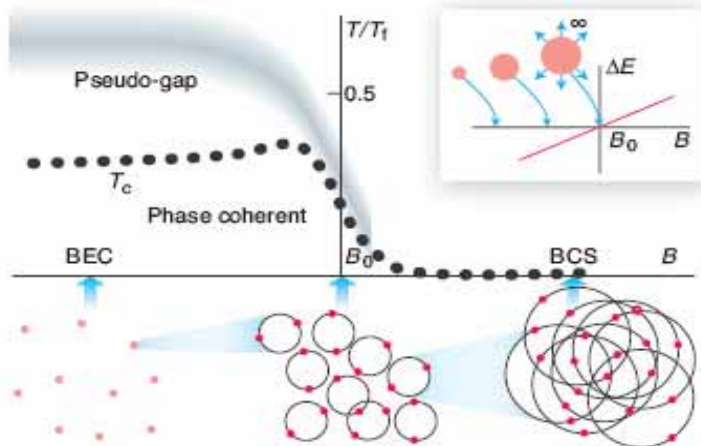
Atom Chip:
原子回路



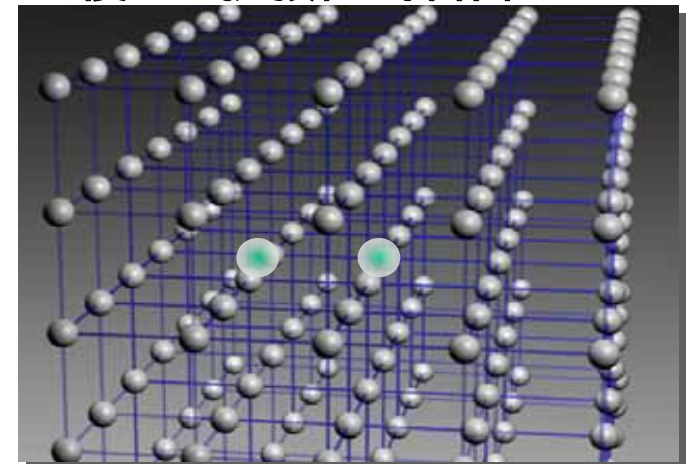
Quantum Simulation:
原子を使ったクリーンな“凝縮系”物理



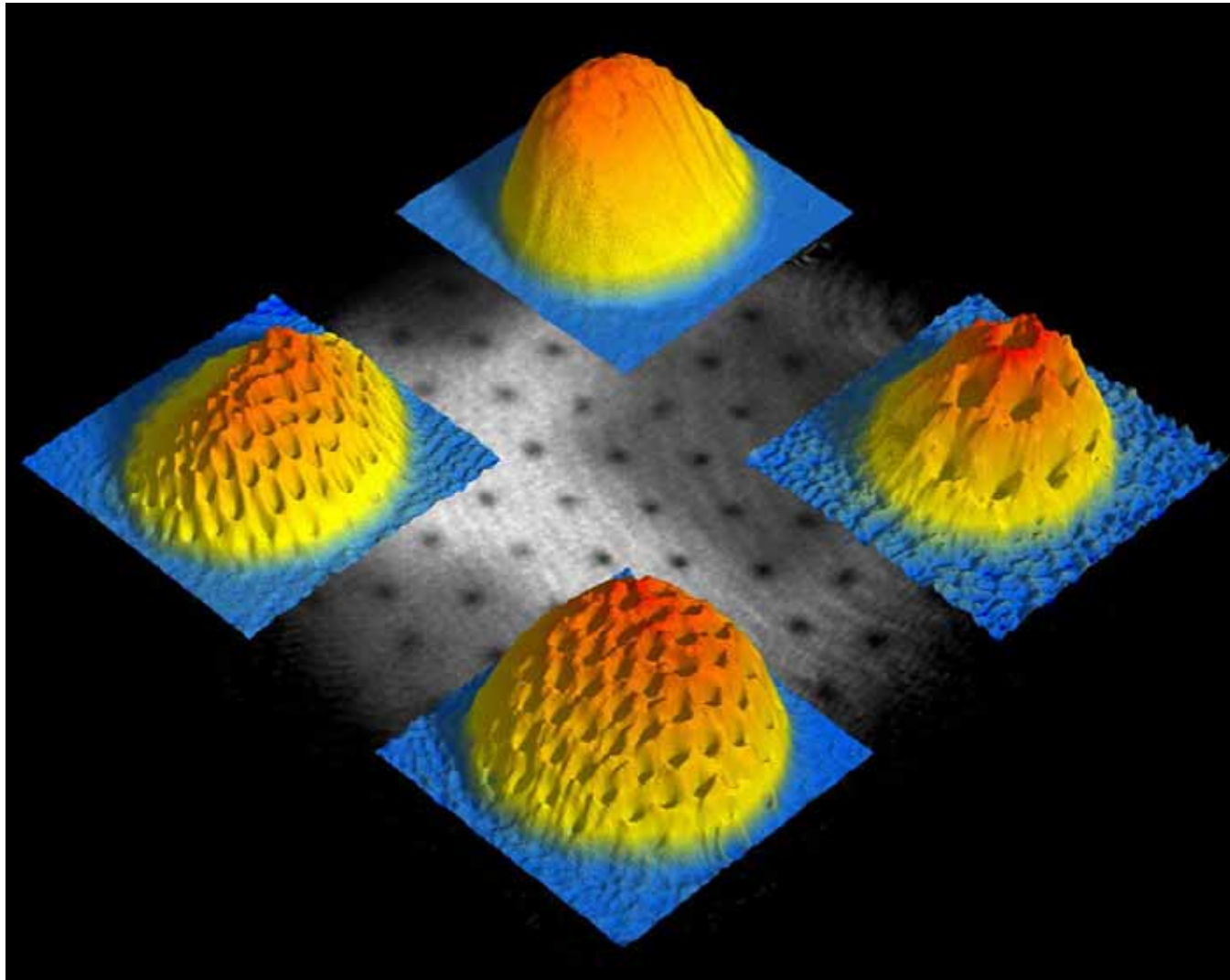
BEC-BCS Crossover:
原子間相互作用の完全なコントロール



Quantum Computation:
優れた拡張性と操作性



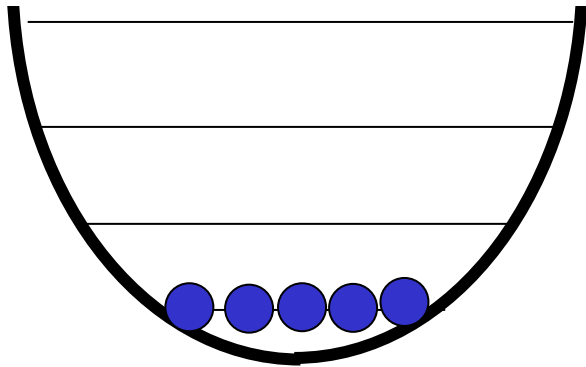
量子渦



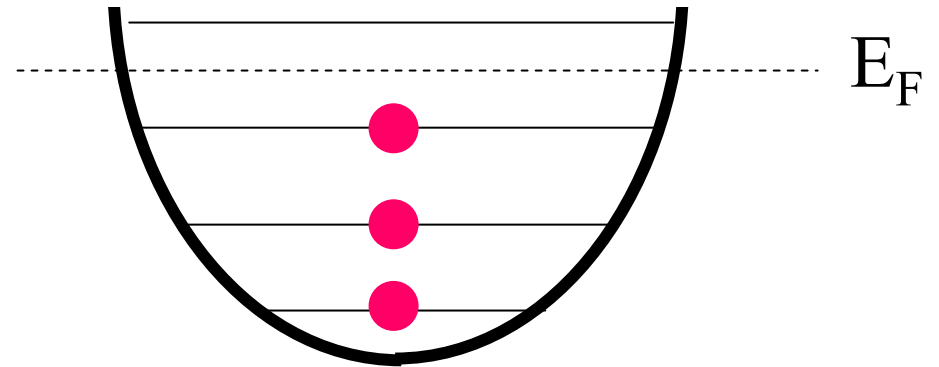
BOSON

VS

FERMION



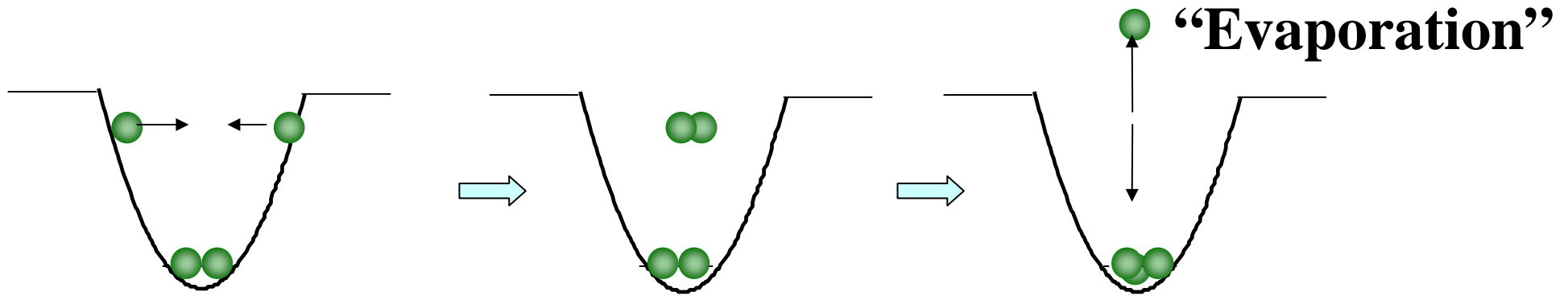
BOSON: $k_B T_C = h \quad (N/1.2)^{1/3}$



FERMION: $k_B T_F = h \quad (6N)^{1/3}$

BOSON vs FERMION

in *Evaporative Cooling*



$$\sigma_{\text{collision}} = 4\pi a_s^2$$

$$\sigma_{\text{collision}} = 8\pi a_s^2$$

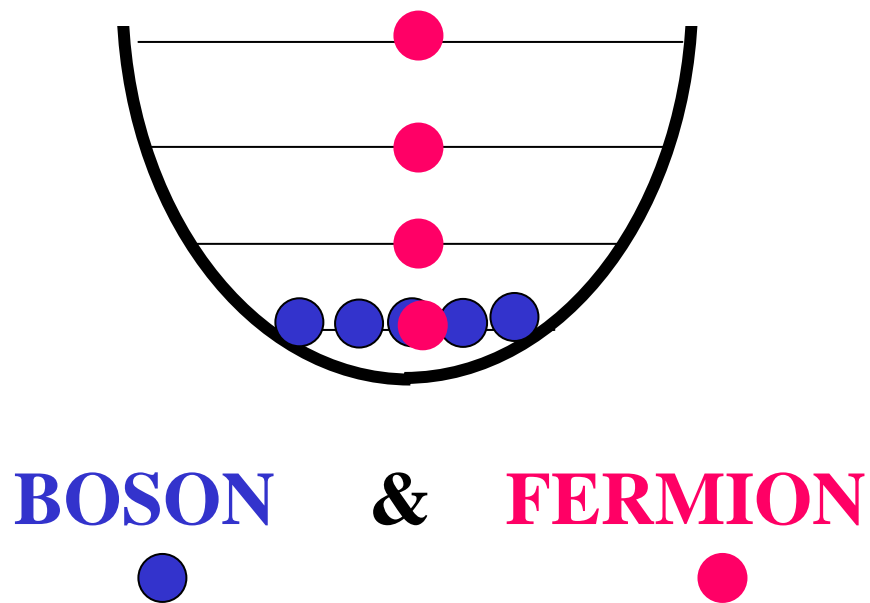
$$\sigma_{\text{collision}} = 0$$

Bosonic Stimulation :

$$R \propto N_{\text{initial}} \times (1 + N_{\text{final}})$$

Pauli Blocking :

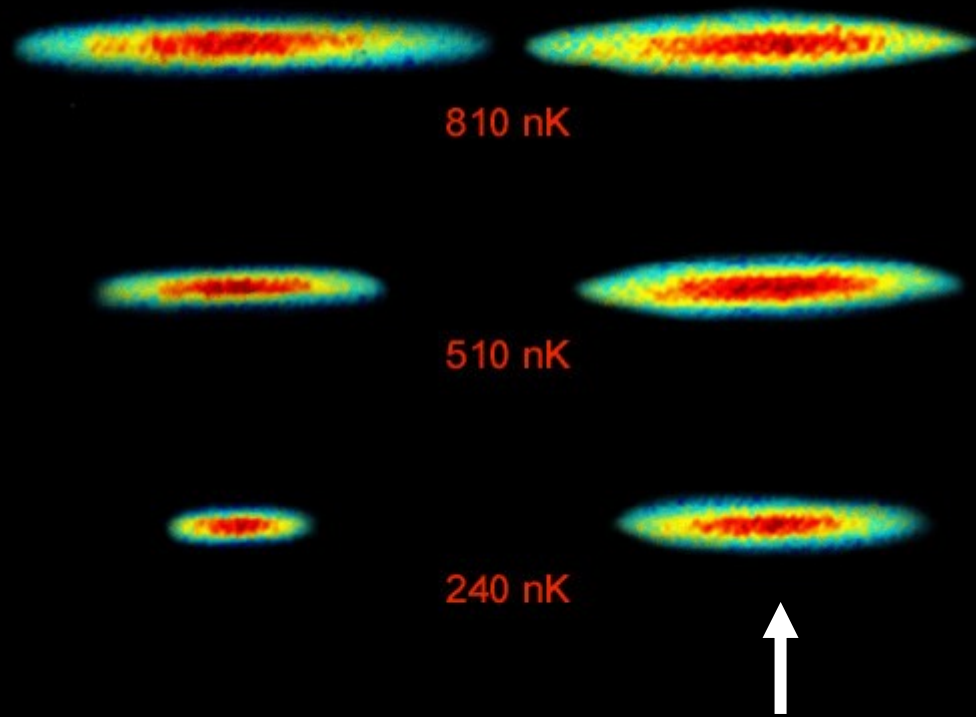
$$R \propto N_{\text{initial}} \times (1 - N_{\text{final}})$$



BEC+Fermi縮退の混合： “協同冷却”

Bosons

Fermions



Fermi pressure

Atomic BCS

$$T_{BCS} \approx 0.3T_F \exp\left(-\frac{\pi}{2k_F |a_s|}\right)$$

典型的な値

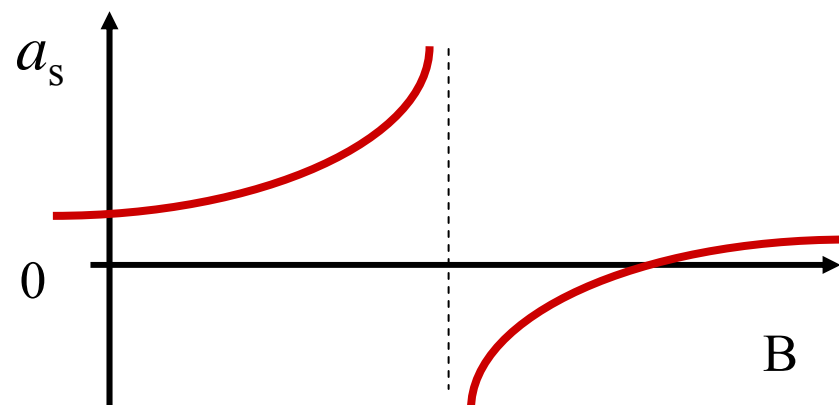
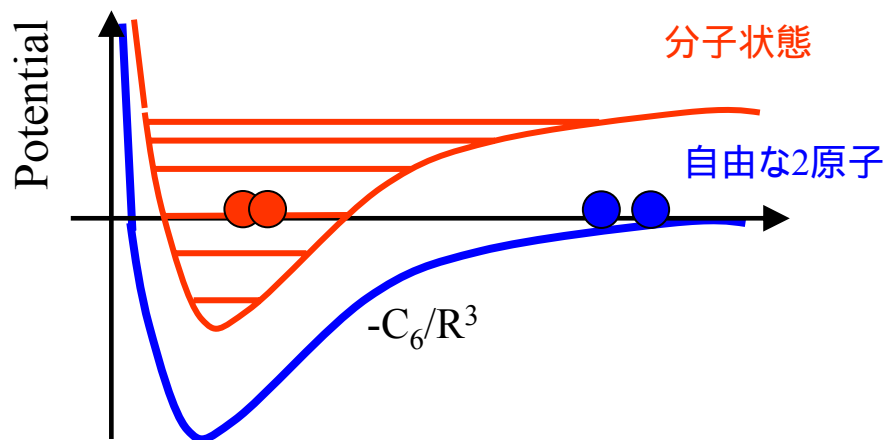
$$T_F \approx 1\mu K, \quad k_F \approx 1/1\mu m, \quad a_s \approx 1nm$$

$$E_F = (3N)^{1/3} \hbar \bar{\omega} = \frac{(\hbar k_F)^2}{2m}$$

Feshbach Resonance

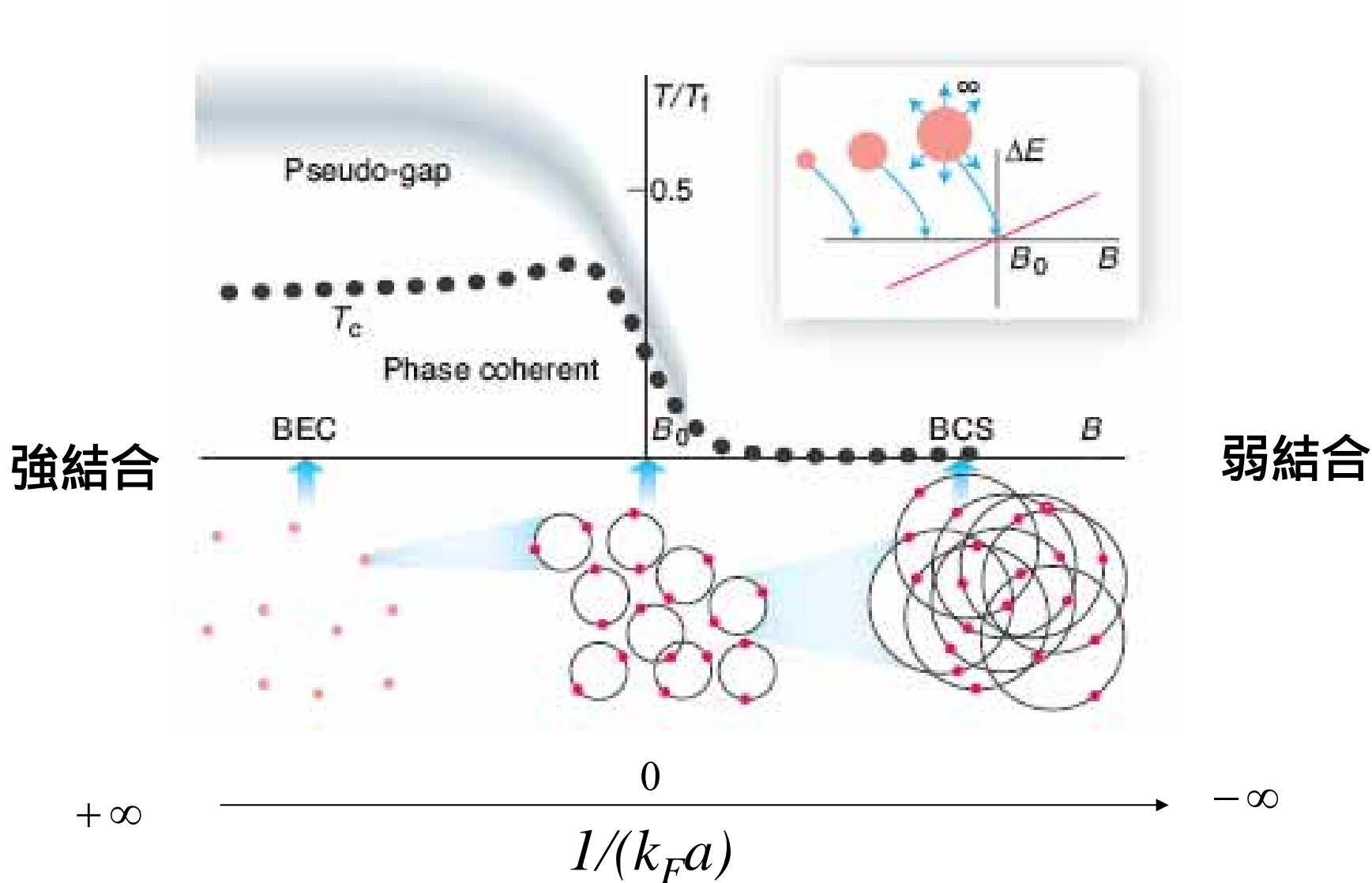
Coupling between “**Open Channel**” and “**Closed Channel**”

→ **Control of a_s** $a_s(B) = a_0 - \frac{C}{B - B_0}$



→ BECの引力崩壊 (Bosenova)
Molecular BEC
BCS

BEC – BCS Crossover



Vortices in BEC-BCS Crossover of ${}^6\text{Li}$ atoms

[M. W. Zwierlein, et al, Nature 435, 1047 (2005)]

${}^6\text{Li}$: $T/T_F > 0.1 \sim 0.2$

“definitive evidence for superfluidity”

BEC side

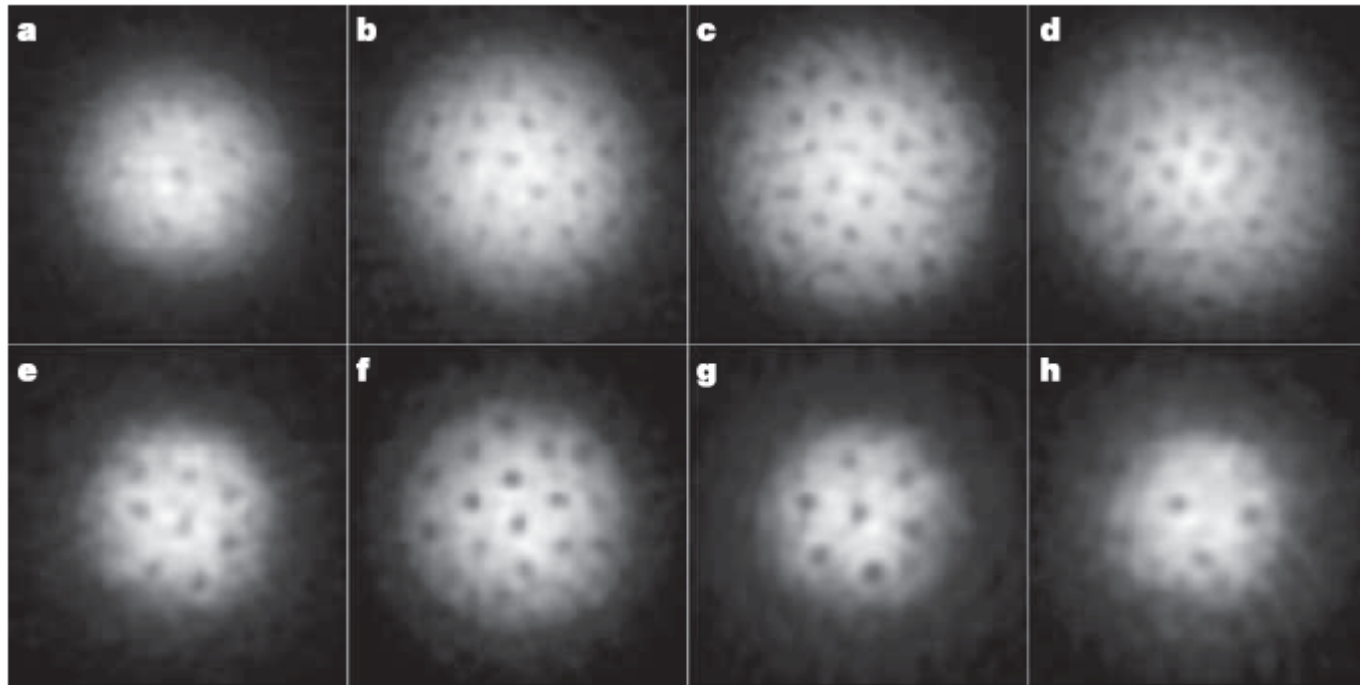


B=745G

766G

792G

812G



833G

843G

853G

863G

BCS side



Atomic Quantum Degenerate Gases

Bose-Einstein condensation

^1H $^4\text{He}^*$ ^7Li
 ^{23}Na ^{41}K ^{85}Rb
 ^{87}Rb ^{133}Cs
 ^{174}Yb ^{52}Cr

Fermi degeneracy

^6Li ^{40}K

Molecular BEC

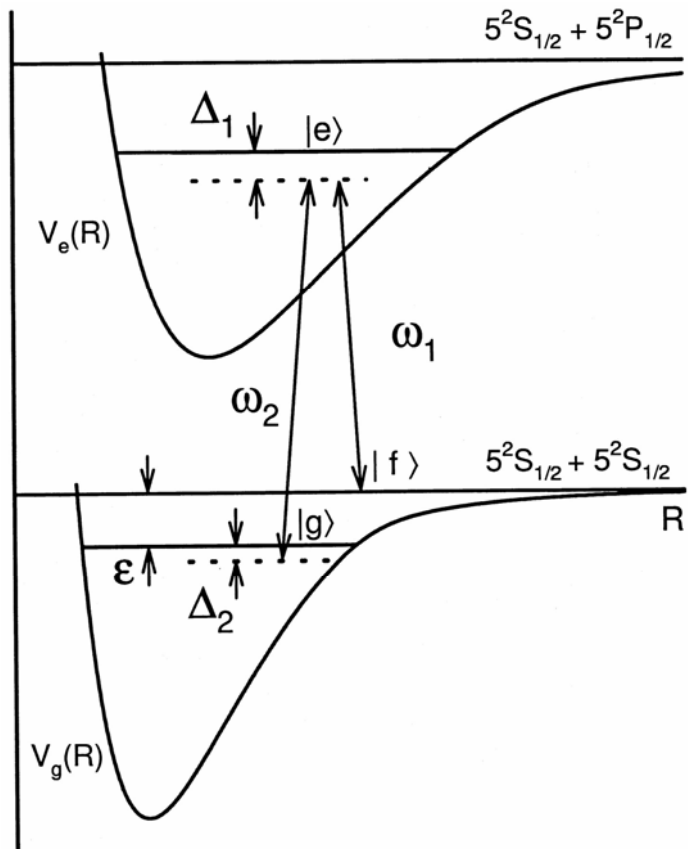
^{23}Na

^6Li ^{40}K

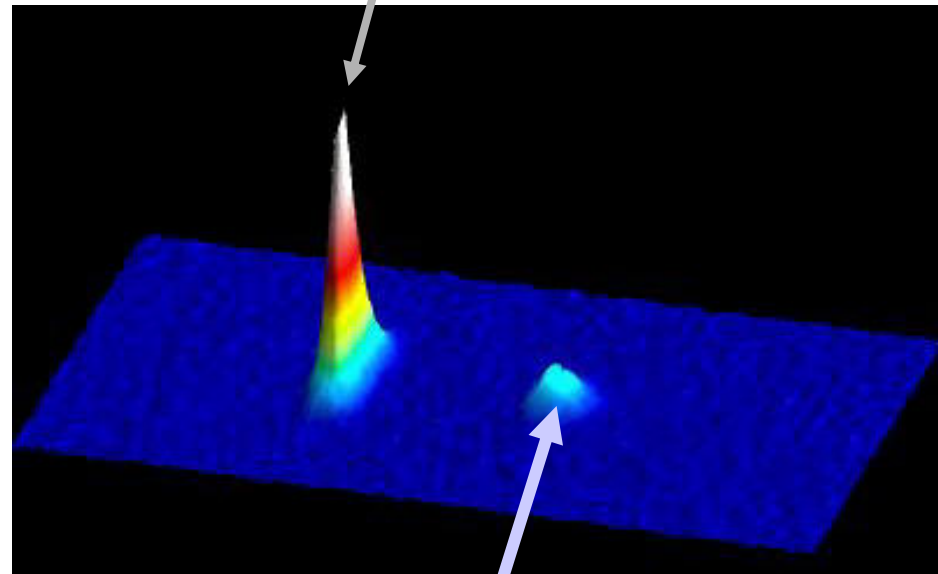
BCS

分子のBEC

Rb分子の光会合



Cs 原子BEC



Cs 分子“BEC”

低温科学A (高橋): 「レーザーによる希薄気体の冷却とボース

レポート問題

- 1) いろいろなレーザー冷却法またはトラップ法の中から、一つを選んで、その原理を述べよ。(50点)
- 2) 希薄気体の量子縮退原子の系の特徴について、液体や固体の系と比較しながら、詳述せよ。(50点)

講義の感想を書いてください。(採点には無関係)

レポート提出場所: 理学部教務係(6号館1F窓口)

レポート提出期限: 7/13日(木)