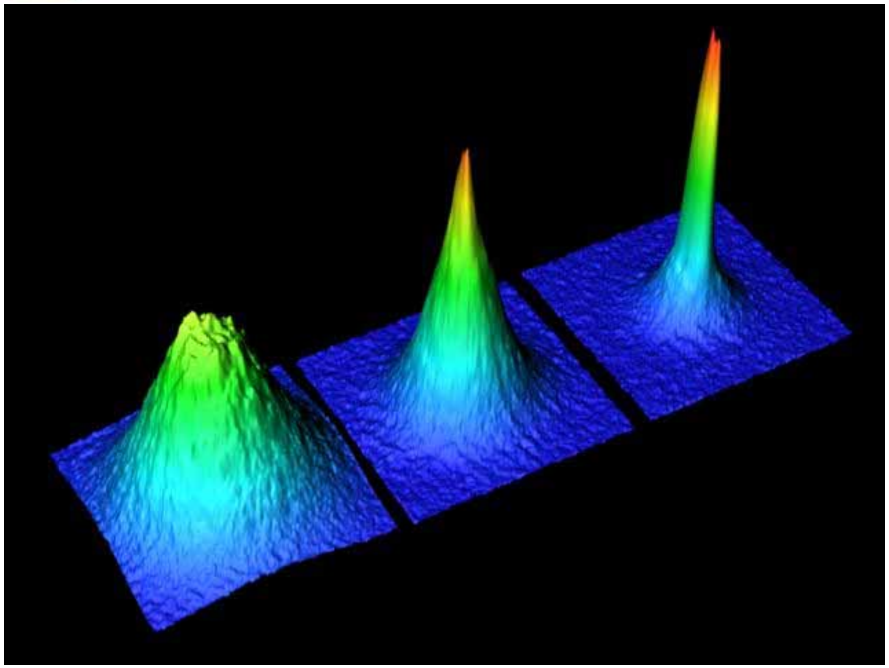
4. 原子気体のボース・アインシュタイン凝縮（BEC）

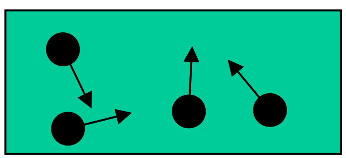
2001 E. Cornell, C. Wieman, W. Ketterle



$$\rho_{PSD} = n\lambda_{dB}^3 = n\left(h/\sqrt{2\pi m_A k_B T}\right)^3$$

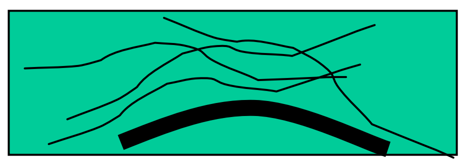
位相空間密度：ρ > 2.612

T_C=100 nK,　n=10^{14}/cm^3



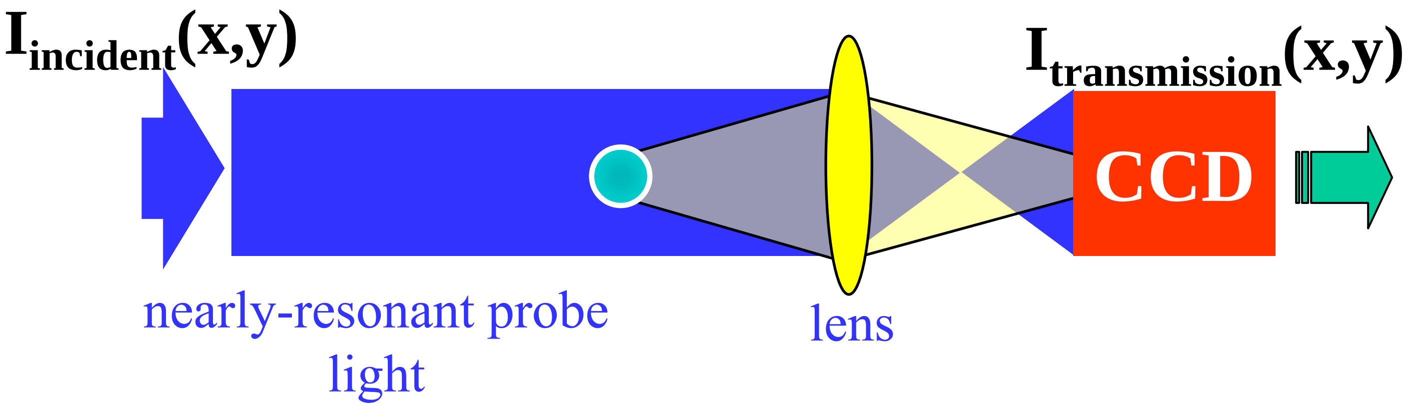
高温：原子はランダムに 熱運動をしています。

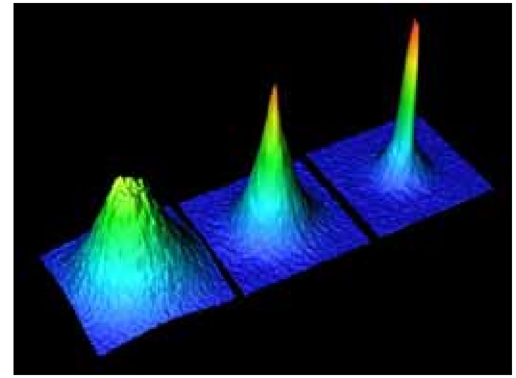
低温：レーザー冷却法により低温になった原子では、波動性が顕著に表れます。



極低温：さらに冷却されるとお互いの波が重なり合い、純粋に量子力学的な相転移が起きます。これがボース・アインシュタイン凝縮(BEC)です。

Optical Imaging

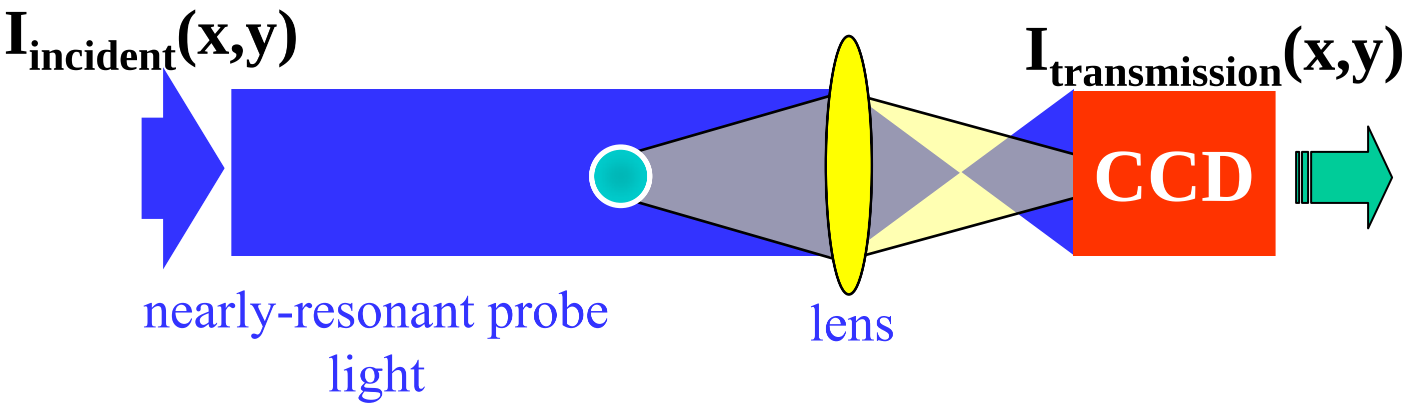


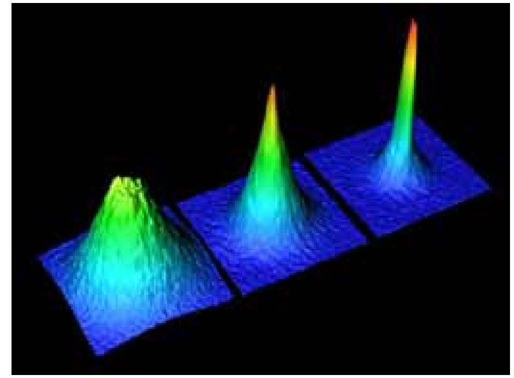


$$I_{transmission}(x,y) = I_{incident}(x,y)\exp(-\sigma_{abs} n(x,y) L)$$

$$\longrightarrow n(x,y) = -\frac{1}{\sigma_{abs} L}\log\left(\frac{I_{transmission}(x,y)}{I_{incident}(x,y)}\right)$$

Optical Imaging





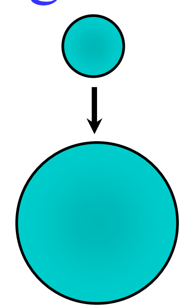
Atom number

$$N = \int_{-\infty}^{+\infty} n(x,y) L\,dx\,dy = -\frac{1}{\sigma_{abs}} \int_{-\infty}^{+\infty} \log\left(\frac{I_{transmission}(x,y)}{I_{incident}(x,y)}\right) dx\,dy$$

Temperature

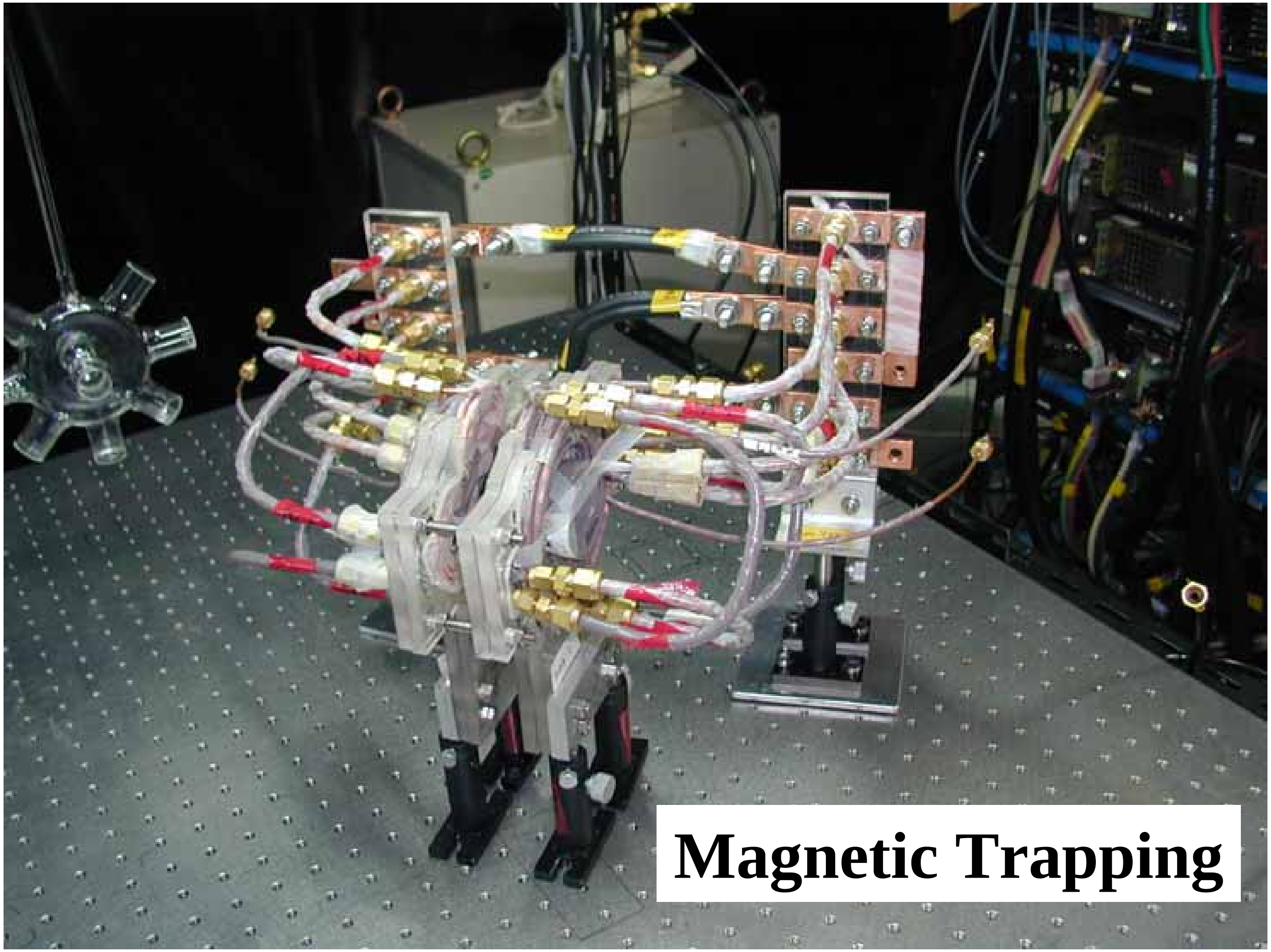
$$T = \frac{M\,(\sigma_{final}^2 - \sigma_{initial}^2)}{k_B\,t^2}$$

Time-of-flight measurement



４－１．ＢＥＣの生成

(i) 磁気トラップ (Magnetic Trap)



$I \sim 300$ A

$B' \sim 200$G/cm

$B'' \sim 160$G/cm^2

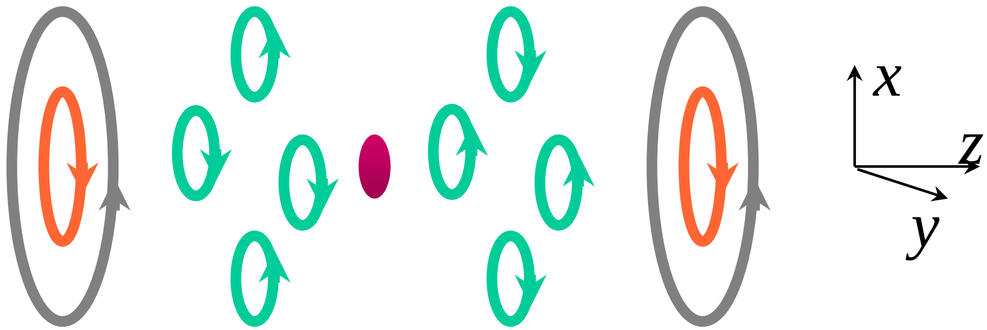
$B_0 \sim 1$ G

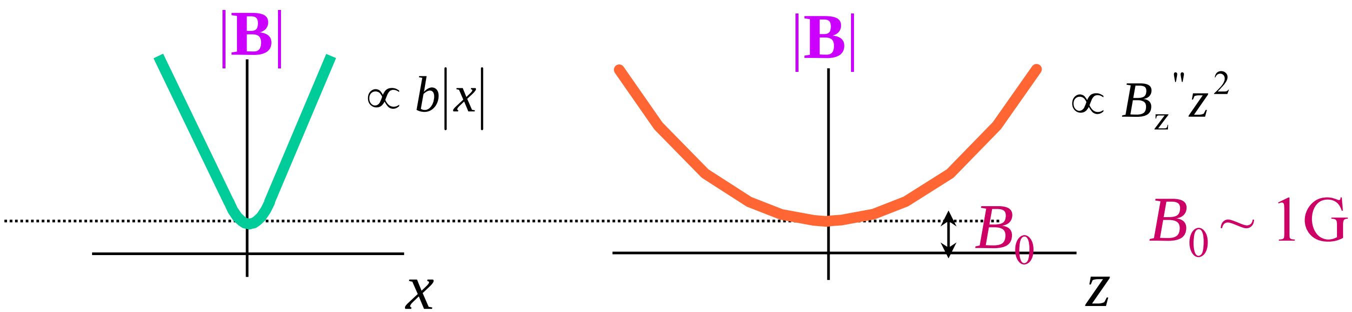
$\omega_r \sim 200$ Hz

$\omega_z \sim 16$ Hz

Ioffe-Pritchard Magnetic Trap

<u>Clover leaf trap</u> (Ioffe-Pritchard trap)



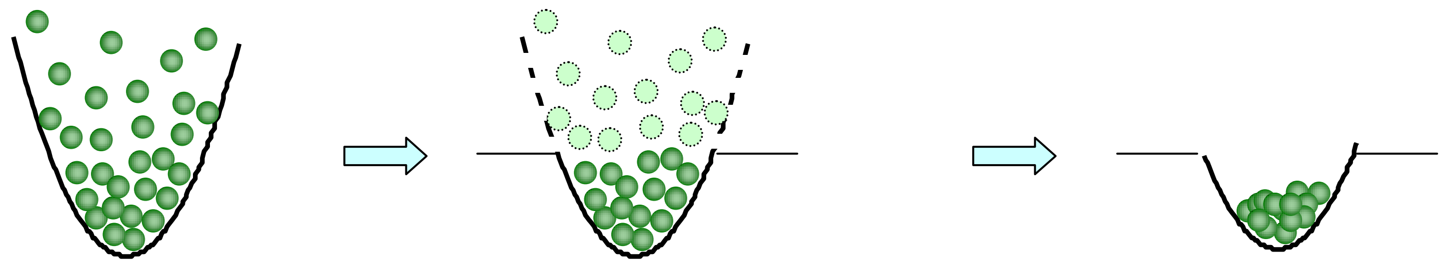


$$\vec{B} = (-bx - B''zx, by + B''zy, B_0 + B''(z^2 - \frac{x^2+y^2}{2})$$

$$\left|\vec{B}\right| \approx B_0 + B''(z^2 - \frac{x^2+y^2}{2}) + \frac{b^2}{2B_0}(x^2+y^2)$$

4－2. Evaporative Cooling(蒸発冷却)

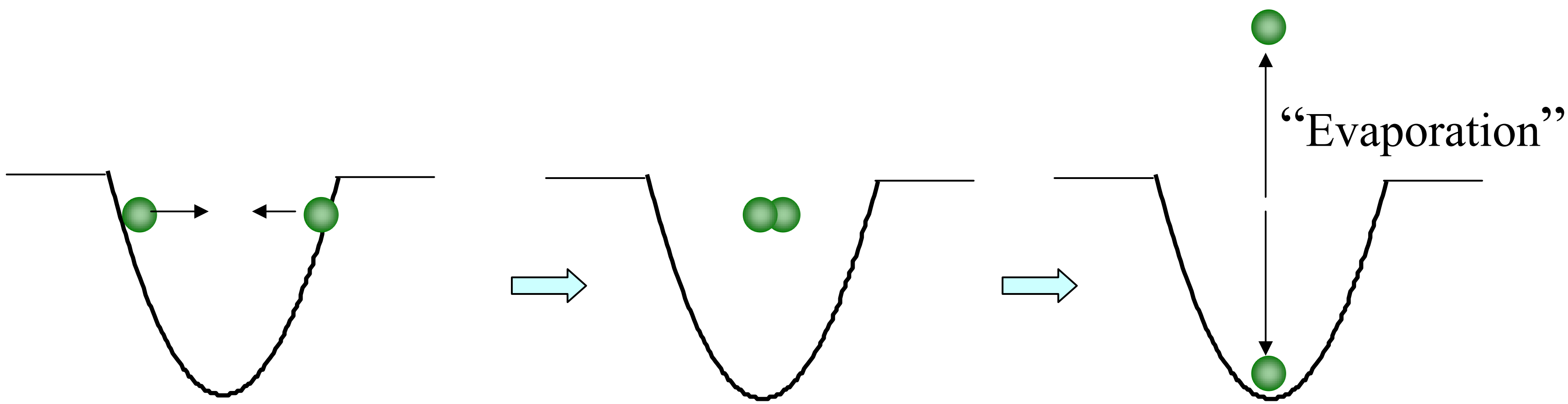
U:high → U:low



N: large
T: high
n: low

Thermalization
(衝突による熱平衡化)

N:small
T:low
n:high



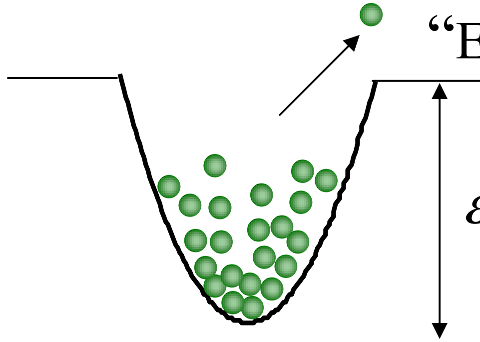
Evaporative Cooling(蒸発冷却)

$\bar{\varepsilon}$: average energy of atoms in trap

ε_{ev} : trap depth

$\eta \equiv \dfrac{\varepsilon_{ev}}{k_B T}$: truncation factor

ε' : average energy of evaporated atoms from trap



“Evaporation” $\varepsilon' \equiv (1+\beta)\times\bar{\varepsilon}$

$-dN$

ε_{ev}

N :total atom number in trap

$E = N\bar{\varepsilon}$:total energy of atoms in trap

<u>Only evaporation considered for atom number loss</u>

$$-dE = -Nd\bar{\varepsilon} - \bar{\varepsilon}dN = \varepsilon' dN = (1+\beta)dN$$

$$\longrightarrow \quad \frac{d\ln\bar{\varepsilon}}{d\ln N} = -\beta$$

For 3D harmonic trap: $\bar{\varepsilon} = (3/2+3/2)k_B T = 3k_B T \quad \longrightarrow \quad \dfrac{d\ln T}{d\ln N} = -\beta$

Evaporative Cooling(蒸発冷却)

loss other than evaporation also included

$$\frac{dN}{N}\Big|_{loss} = -\frac{dt}{\tau_{loss}} \quad , \frac{dN}{N}\Big|_{ev} = -\frac{dt}{\tau_{ev}} \longrightarrow \frac{dN|_{ev}}{dN|_{ev} + dN|_{loss}} = \frac{\tau_{loss}}{\tau_{loss} + \tau_{ev}}$$

$$\longrightarrow \frac{d\ln T}{d\ln N} = -\beta \frac{\tau_{loss}}{\tau_{loss} + \tau_{ev}} \equiv -\beta'$$

If η >>1, then

$$\frac{dN}{dt}\Big|_{ev} = -Nn(0)\sigma\bar{v}\left(\frac{\varepsilon_{ev}}{k_B T}\right)\exp\left(-\frac{\varepsilon_{ev}}{k_B T}\right)$$

$n(0)$: peak atom density

$\bar{v} = \left(\frac{8k_B T}{\pi m}\right)^{1/2}$: average atom velocity

$$\longrightarrow \frac{1}{\tau_{ev}} = \frac{1}{\tau_{el}}\left(\frac{\varepsilon_{ev}}{\sqrt{2}k_B T}\right)\exp\left(-\frac{\varepsilon_{ev}}{k_B T}\right) \quad , \frac{1}{\tau_{el}} \equiv n(0)\sqrt{2}\bar{v}\sigma$$

If $\varepsilon' \approx \varepsilon_{ev} \longrightarrow$

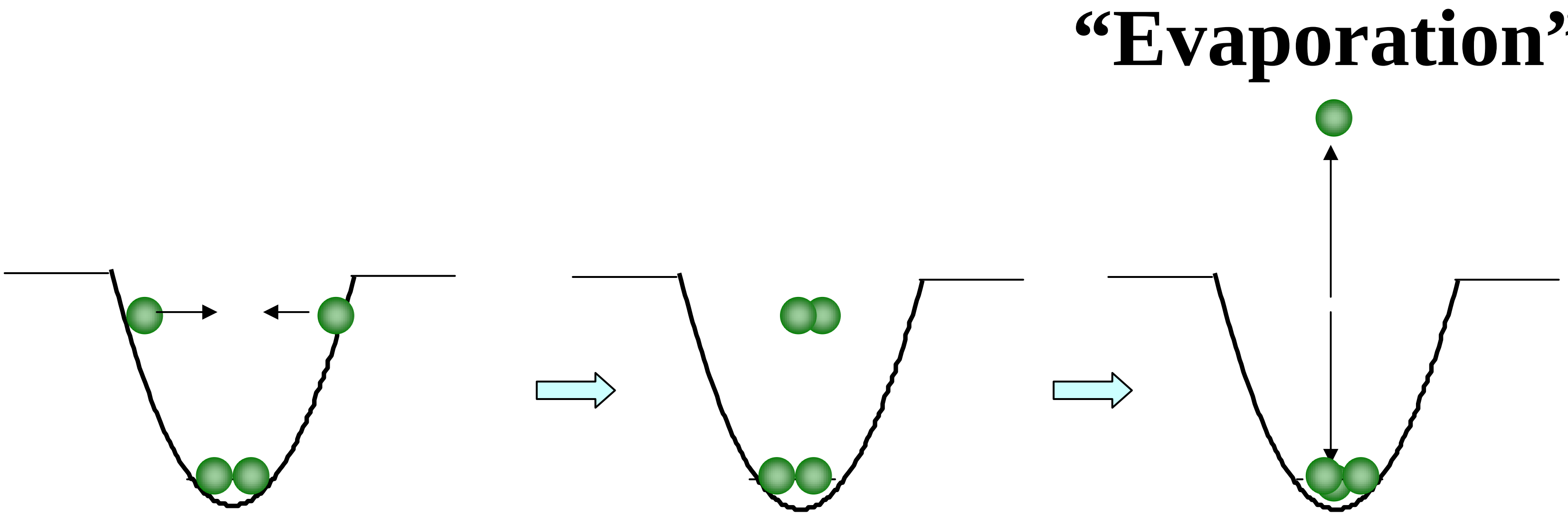
$$\frac{d\ln T}{d\ln N} = \left(\frac{\varepsilon_{ev}}{\bar{\varepsilon}} - 1\right)\left(1 + \frac{\tau_{el}}{\tau_{loss}}\frac{\sqrt{2}k_B T}{\varepsilon_{ev}}\exp\left(\frac{\varepsilon_{ev}}{k_B T}\right)\right)^{-1}$$

$$\frac{d\ln \rho_{PSD}}{d\ln N} = -3\beta' + 1 \qquad \frac{d\ln \tau_{el}}{d\ln N} = \beta' - 1 > 0$$

(runaway evaporation)

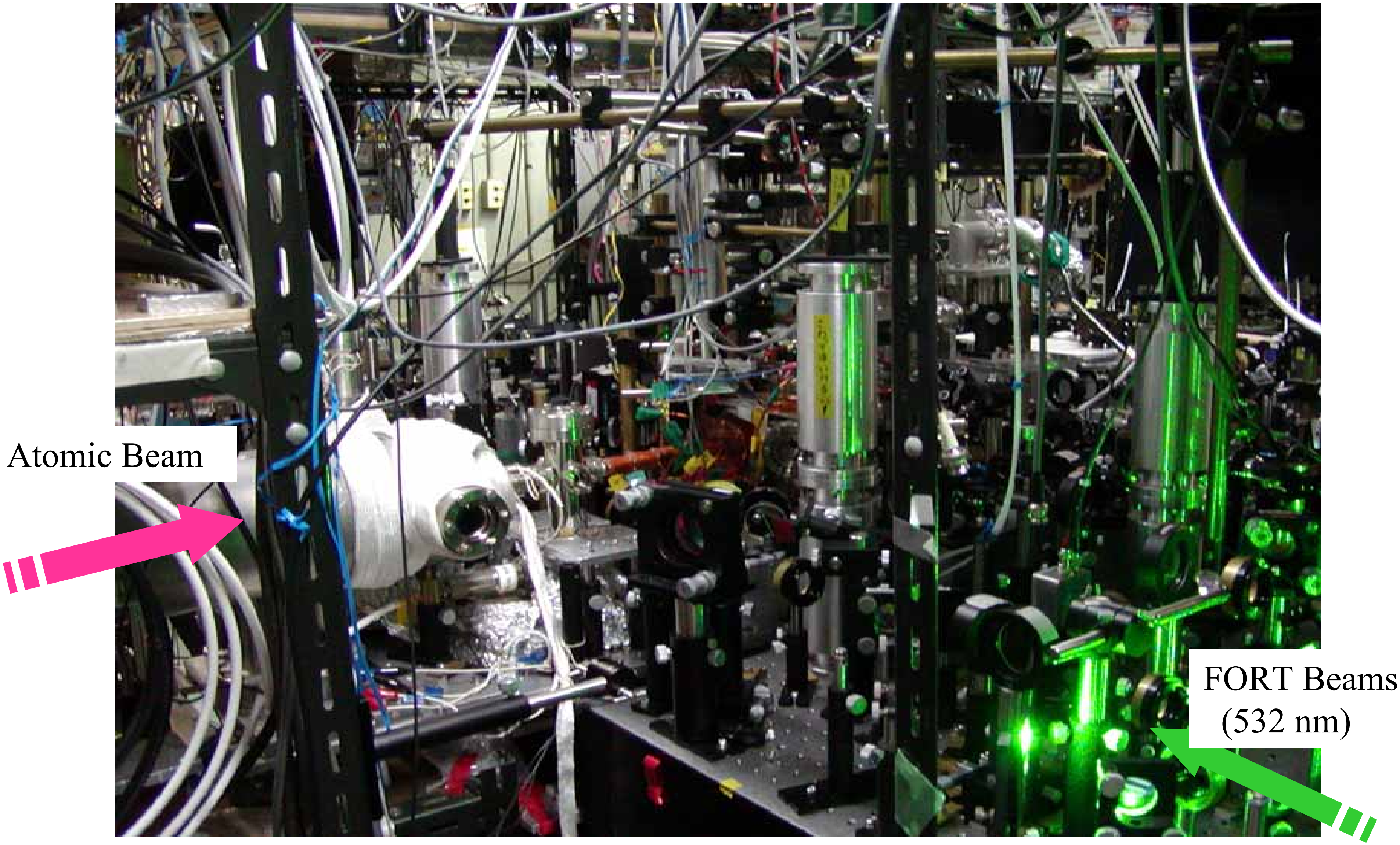
Evaporative Cooling(蒸発冷却)

$$T \approx T_C$$



Bosonic Stimulation : $R \propto N_{initial} \times (1 + N_{final})$

実験装置



Atomic BEC

1995	^{87}Rb, ^{23}Na, ^{7}Li
1998	^{1}H
2000	^{85}Rb
2001	^{41}K ^{4}He*
2003	^{133}Cs ^{174}Yb
2005	^{52}Cr

4－2．基本的性質

巨視的な数の原子の波動関数: $\Psi(r_1,\cdots,r_N)=\prod_{i=1}^{N}\phi(r_i)$

single-particle wavefunction $\phi(r)$ Normalization: $\int dr|\phi(r)|^2=1$

$\longrightarrow$ <u>Condensate wavefunction:</u> **“**order parameter”

$$\Phi(r)\equiv N^{1/2}\phi(r) \qquad n(r)=|\Phi(r)|^2, N=\int dr|\Phi(r)|^2$$

<u>ラグランジュの未定乗数法</u>

$$\delta E(\Phi^*,\Phi)-\mu N(\Phi^*,\Phi)=0$$ μ :chemical potential

$$\longrightarrow \quad \left(-\frac{\hbar^2}{2m}\Delta+V(r)+U_0|\Phi(r)|^2\right)\Phi(r)=\mu\Phi(r) \qquad U_0=\frac{4\pi\hbar^2 a_s}{m}$$

“Gross-Pitaevskii 方程式”

a_s **:散乱長**

- **a_s>0:repulsive(斥力)** $\longrightarrow$ **安定**
- **a_s <0:attractive(引力)** $\longrightarrow$ **不安定**:N<N_c

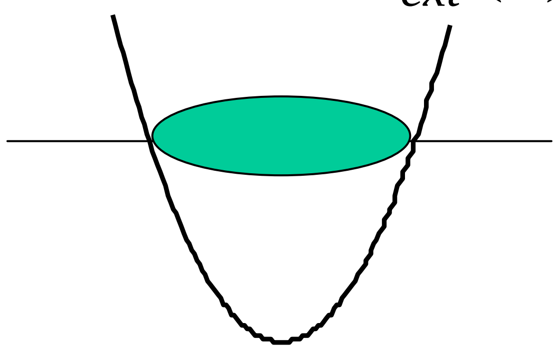
原子気体BEC：Thomas-Fermi近似

$$\mu\Phi = \left(-\frac{\hbar^2}{2m}\Delta + V_{ext} + U_0 |\Phi|^2 \right)\Phi$$

$$\longrightarrow \quad \mu\Phi = \left(V_{ext} + U_0 n(r) \right)\Phi$$

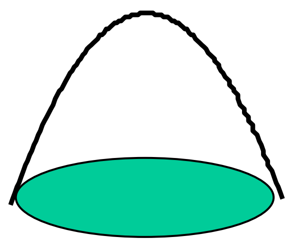
$$\longrightarrow \quad n(r) = (\mu - V_{ext}) / U_0$$

$V_{ext}(r)$

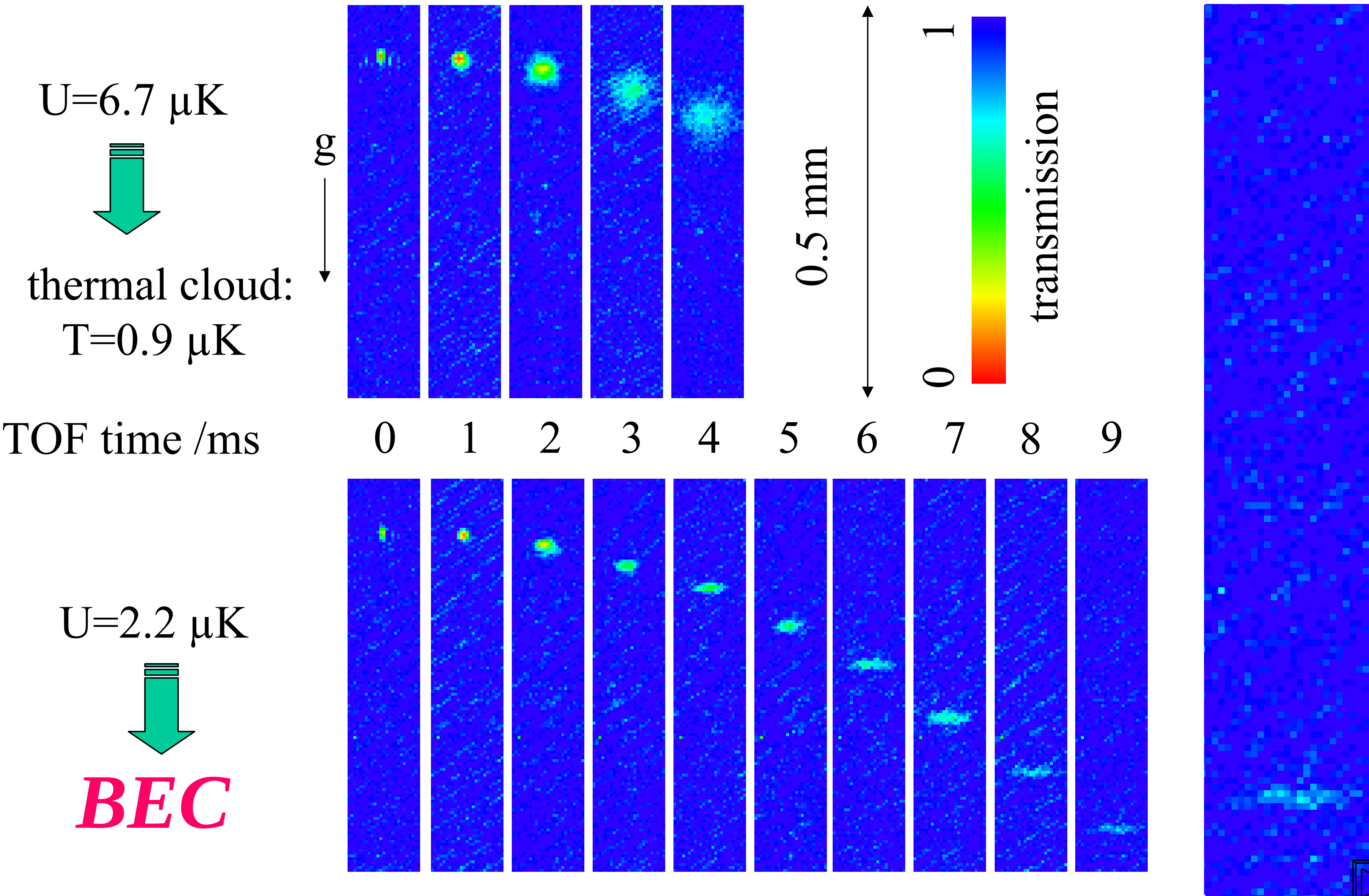


μ

$n(r)$:密度分布



BEC


U=6.7 μK
thermal cloud:
T=0.9 μK
g
0.5 mm
1
0
transmission
TOF time /ms
0 1 2 3 4 5 6 7 8 9
U=2.2 μK
BEC

非等方的な広がり

BEC in Thomas-Fermi regime: $\omega_x = \omega_y \equiv \omega_r >> \omega_z$

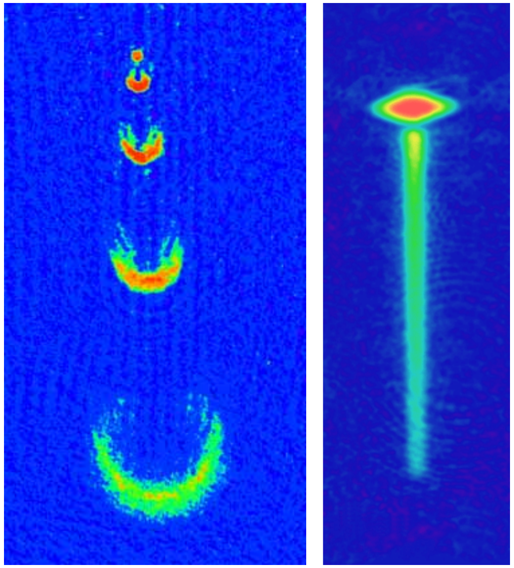
$$n_{TF}(r,t) = \frac{\mu}{g}\left[1 - \left(\frac{r^2}{d_r^2(t)} + \frac{z^2}{d_z^2(t)}\right)\right] \qquad d_{r,z}(0) \equiv \sqrt{\frac{2\mu}{m\omega_{r,z}{}^2}}$$

$$\varepsilon_{TF}(t) \equiv \frac{d_r(t)}{d_z(t)}$$ ：アスペクト比

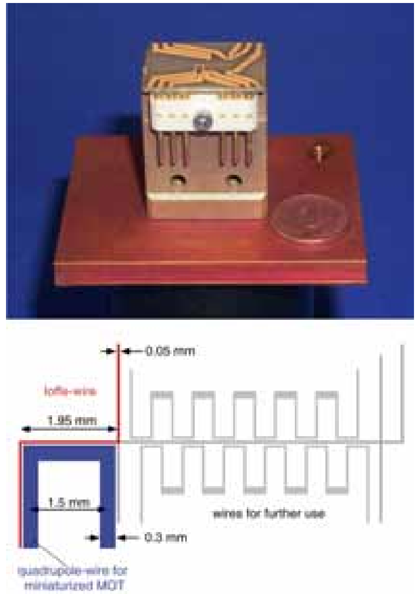
$$\longrightarrow \quad \varepsilon_{TF}(0) = \frac{\omega_z}{\omega_r} << 1 \qquad \varepsilon_{TF}(t) \xrightarrow{t\to\infty} \frac{2}{\pi}\frac{\omega_r}{\omega_z} >> 1$$

4－3. 様々な応用

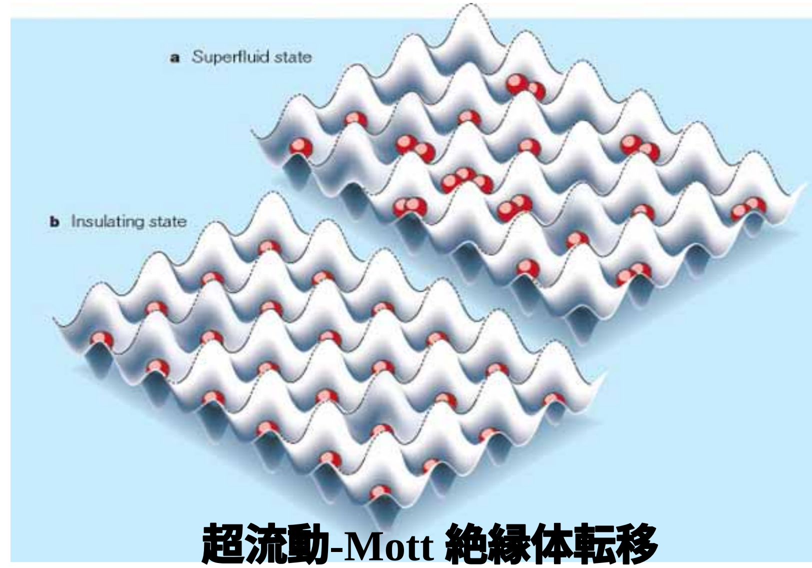
Atom Laser：
コヒーレントな物質波



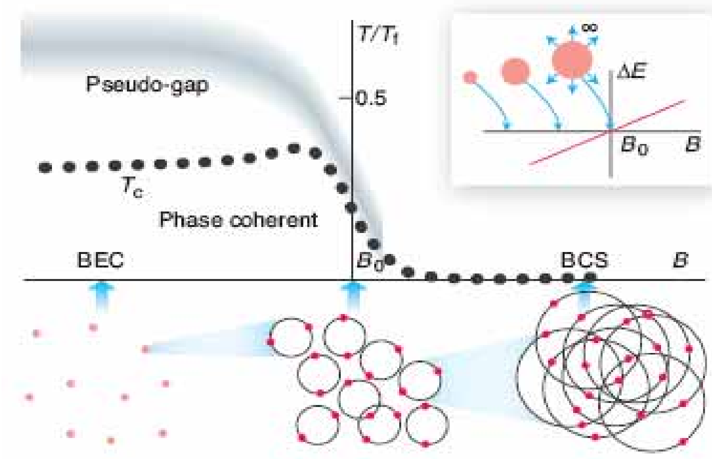
Atom Chip:
原子回路



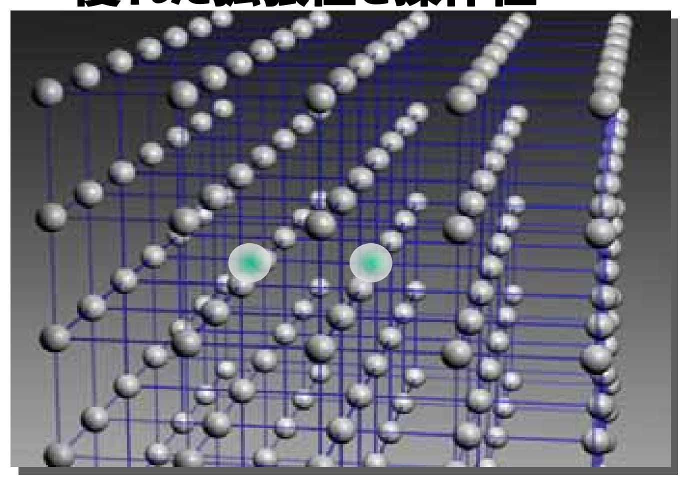
Quantum Simulation:
原子を使ったクリーンな“凝縮系”物理



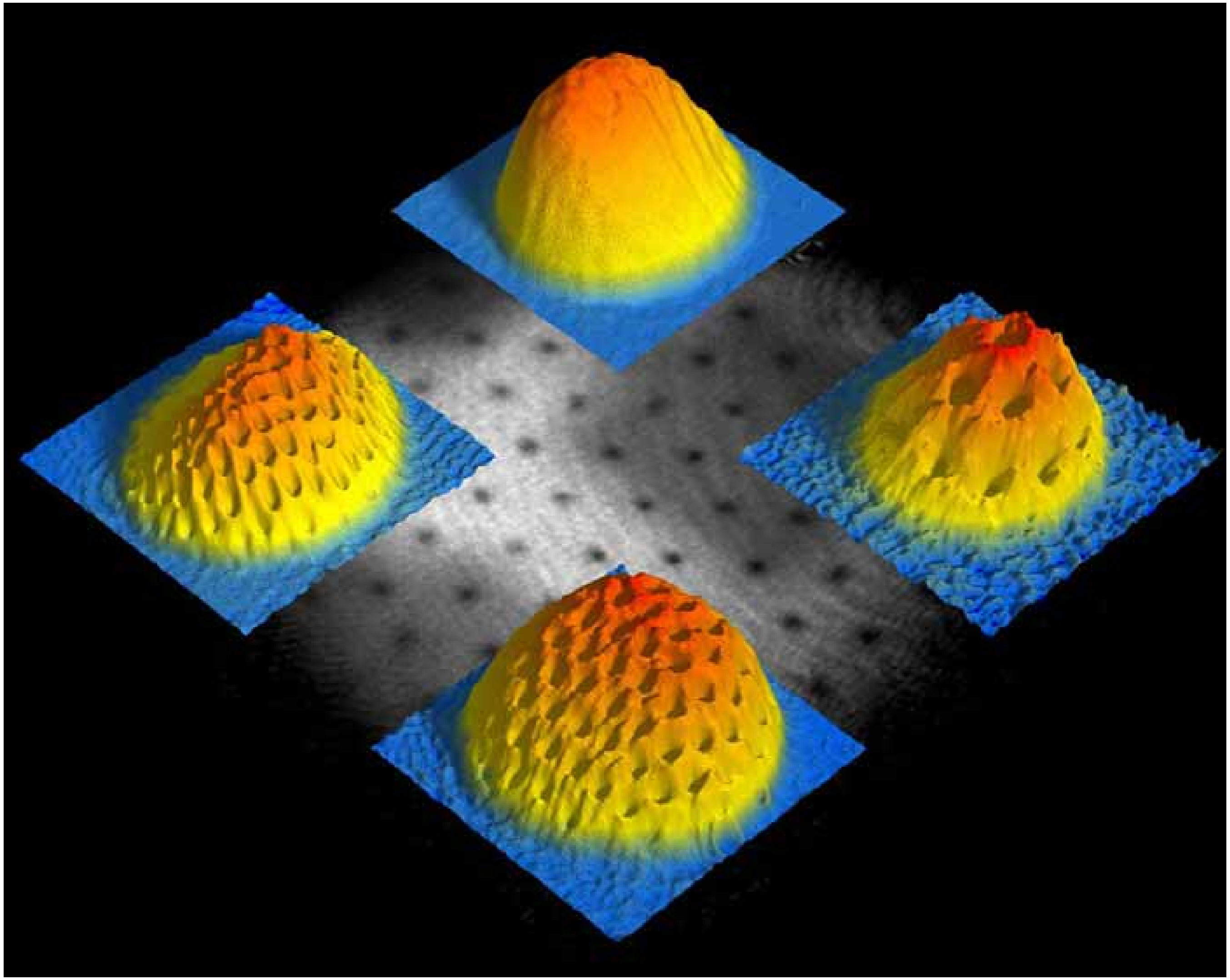
BEC-BCS Crossover:
原子間相互作用の完全なコントロール



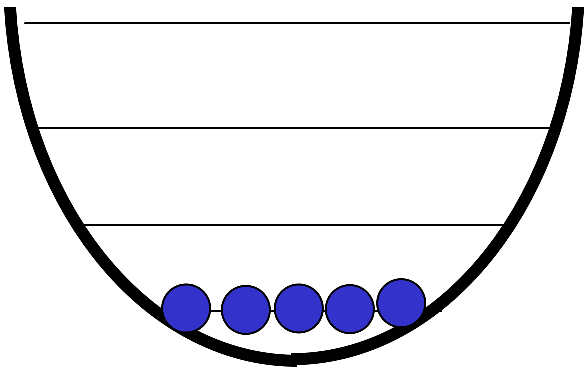
Quantum Computation：
優れた拡張性と操作性

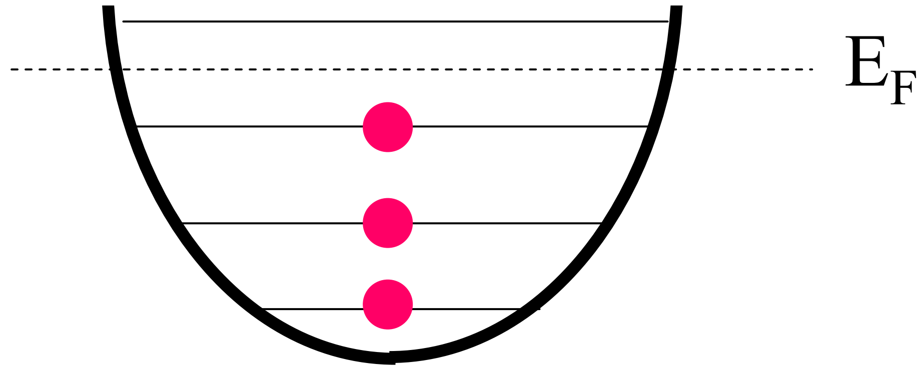


量子渦



BOSON vs FERMION



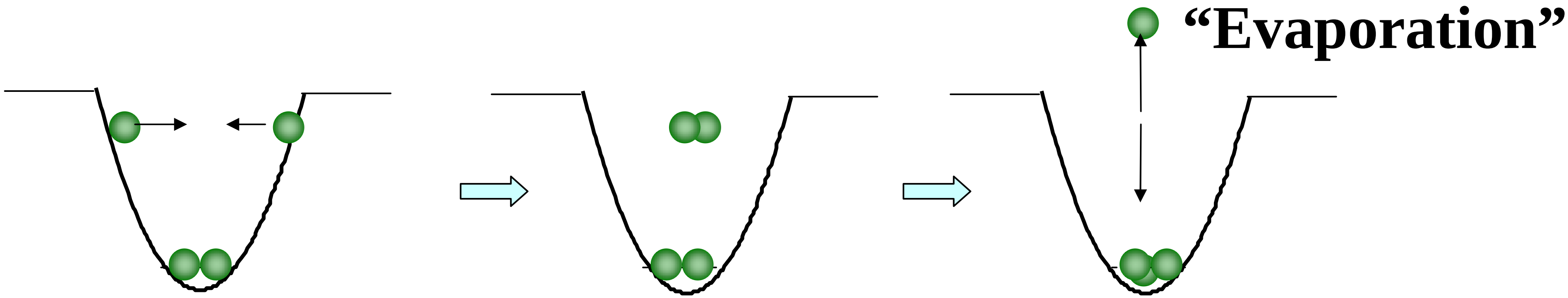


BOSON: $k_B T_C = h\nu \, (N/1.2)^{1/3}$

FERMION: $k_B T_F = h\nu \, (6N)^{1/3}$

BOSON vs FERMION

in *Evaporative Cooling*



$$\sigma_{collision} = 4\pi a_s^2$$

$$\sigma_{collision} = 8\pi a_s^2$$

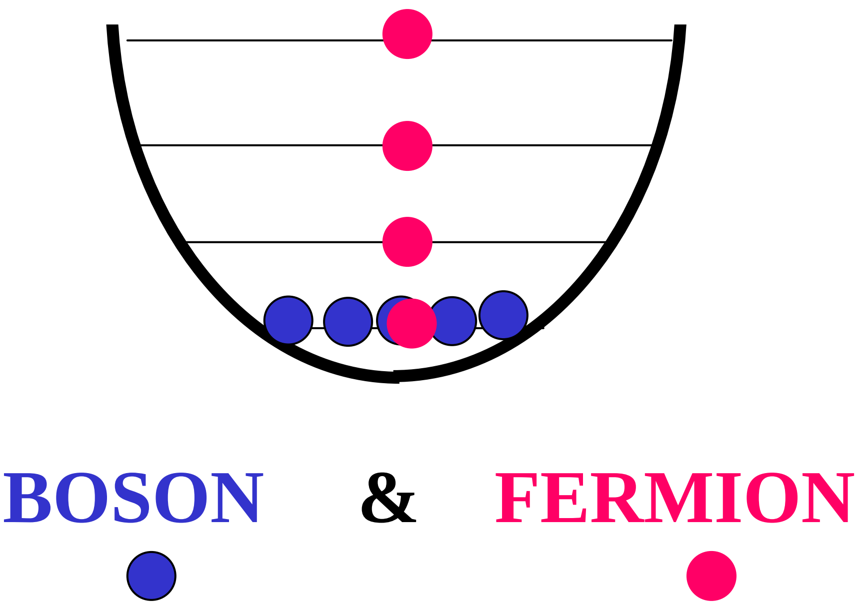
$$\sigma_{collision} = 0$$

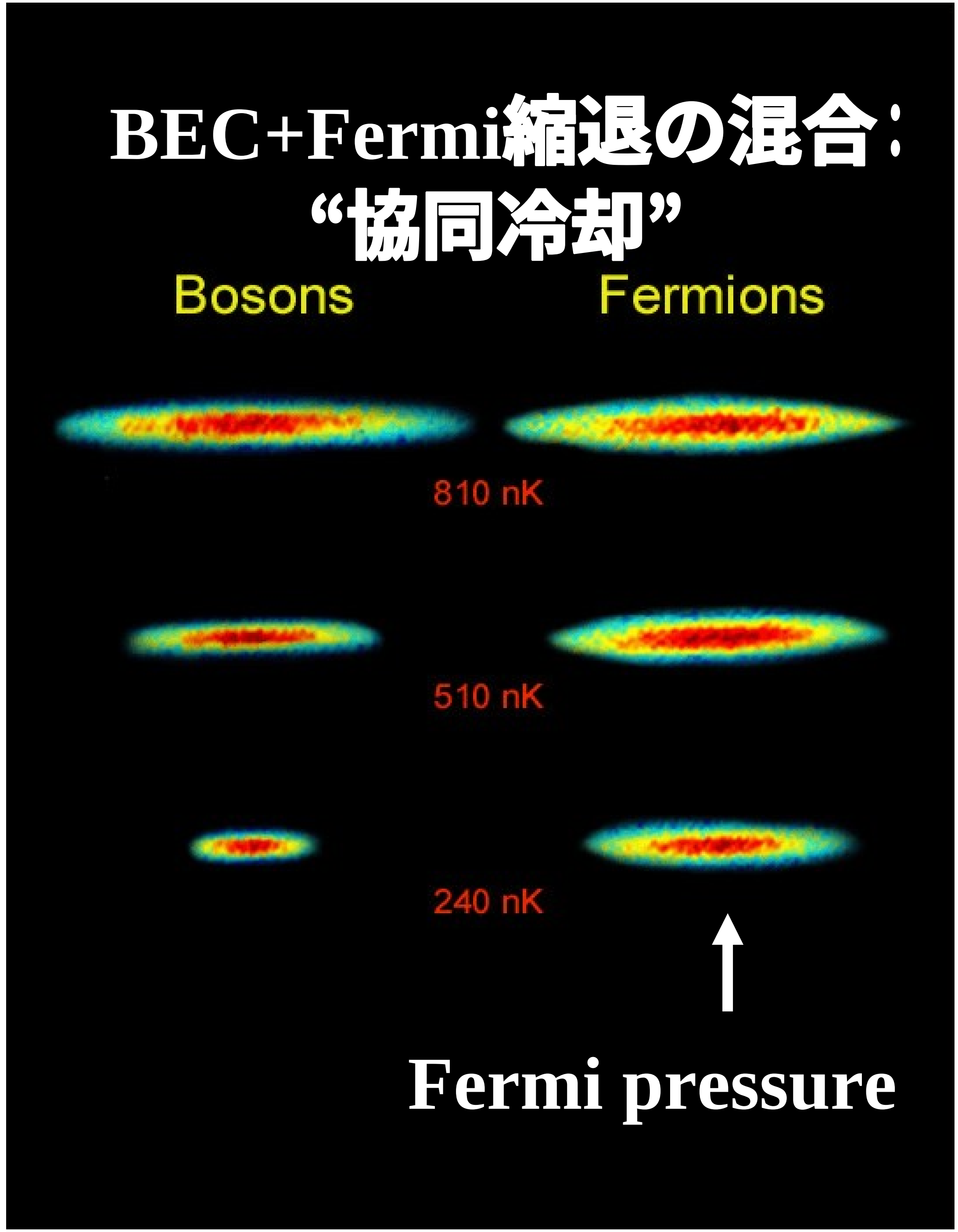
Bosonic Stimulation **:**

$$R \propto N_{initial} \times (1 + $$

Pauli Blocking **:**

$$R \propto N_{initial} \times (1 - N_{final})$$


BOSON
&
FERMION


BEC+Fermi縮退の混合：
“協同冷却”
Bosons
Fermions
810 nK
510 nK
240 nK
Fermi pressure

Atomic BCS

$$T_{BCS} \approx 0.3 T_F \exp\left(-\frac{\pi}{2k_F|a_s|}\right)$$

典型的な値

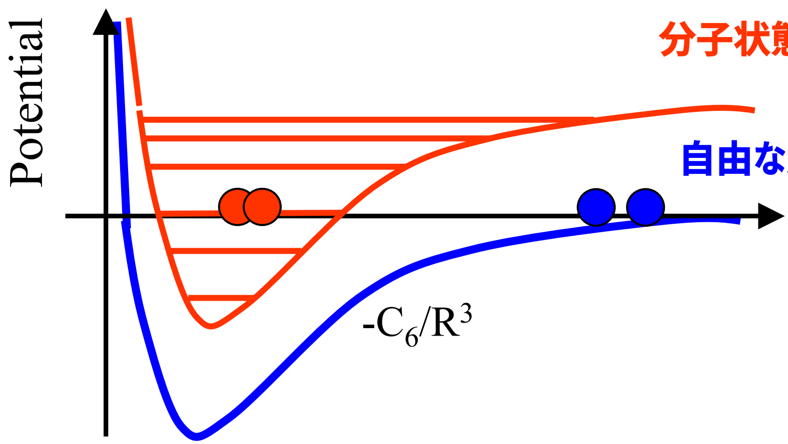
$$T_F \approx 1\mu K, \qquad k_F \approx 1/1\mu m, \qquad a_s \approx 1nm$$

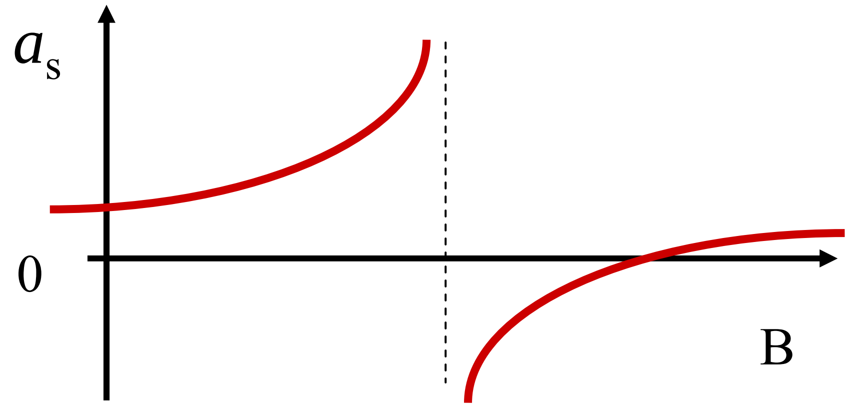
$$\mathrm{E_F} = (3\mathrm{N})^{1/3}\hbar\overline{\omega} = \frac{(\hbar k_F)^2}{2m}$$

Feshbach Resonance

Coupling between "**Open Channel**" and "**Closed Channel**"

→ Control of a_s $$a_s(B) = a_0 - \frac{C}{B - B_0}$$

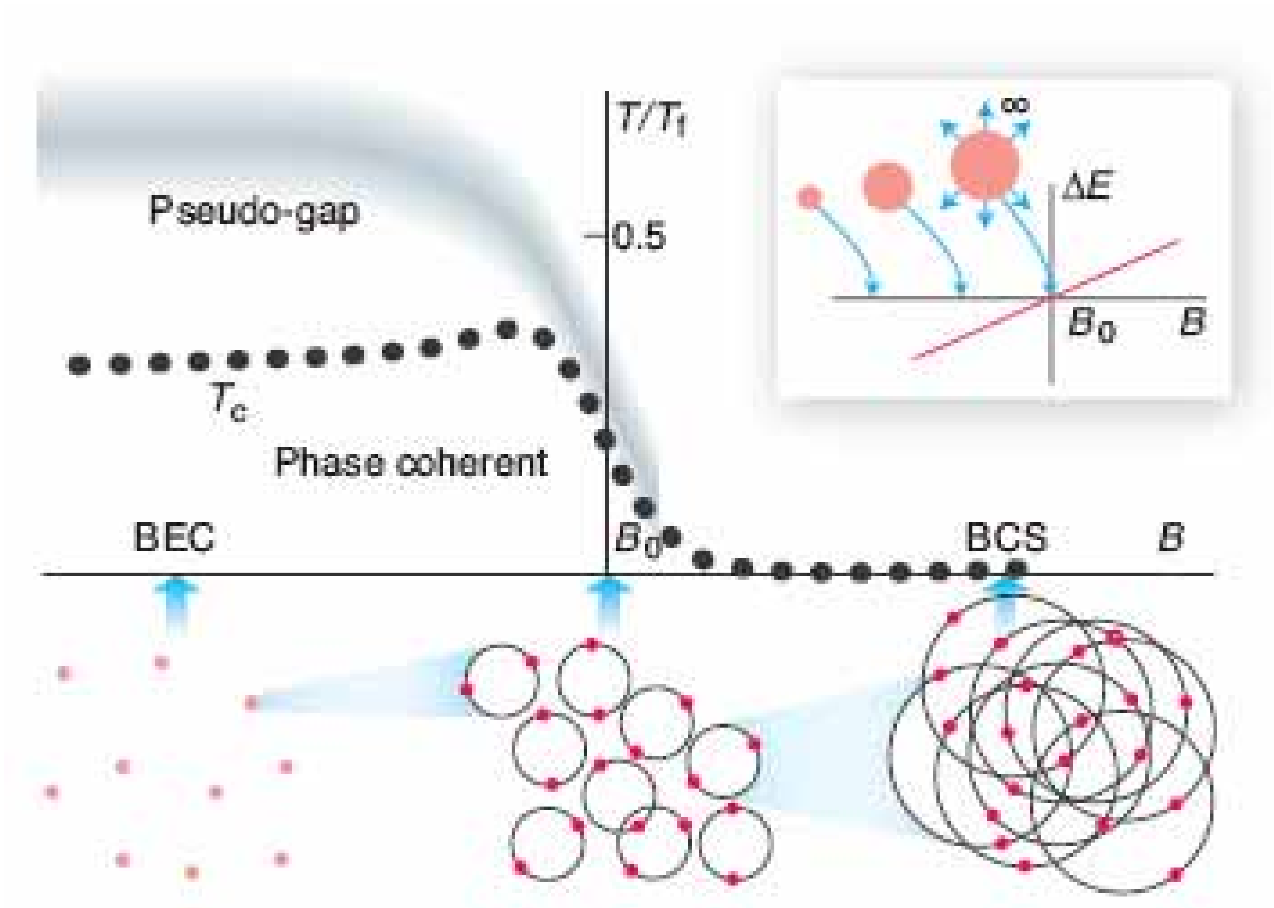




→ BECの引力崩壊 (Bosenova)
Molecular BEC
BCS

BEC – BCS Crossover

強結合



弱結合

0

$+\infty$ → $-\infty$

$1/(k_F a)$

Vortices in BEC-BCS Crossover of ^{6}Li atoms

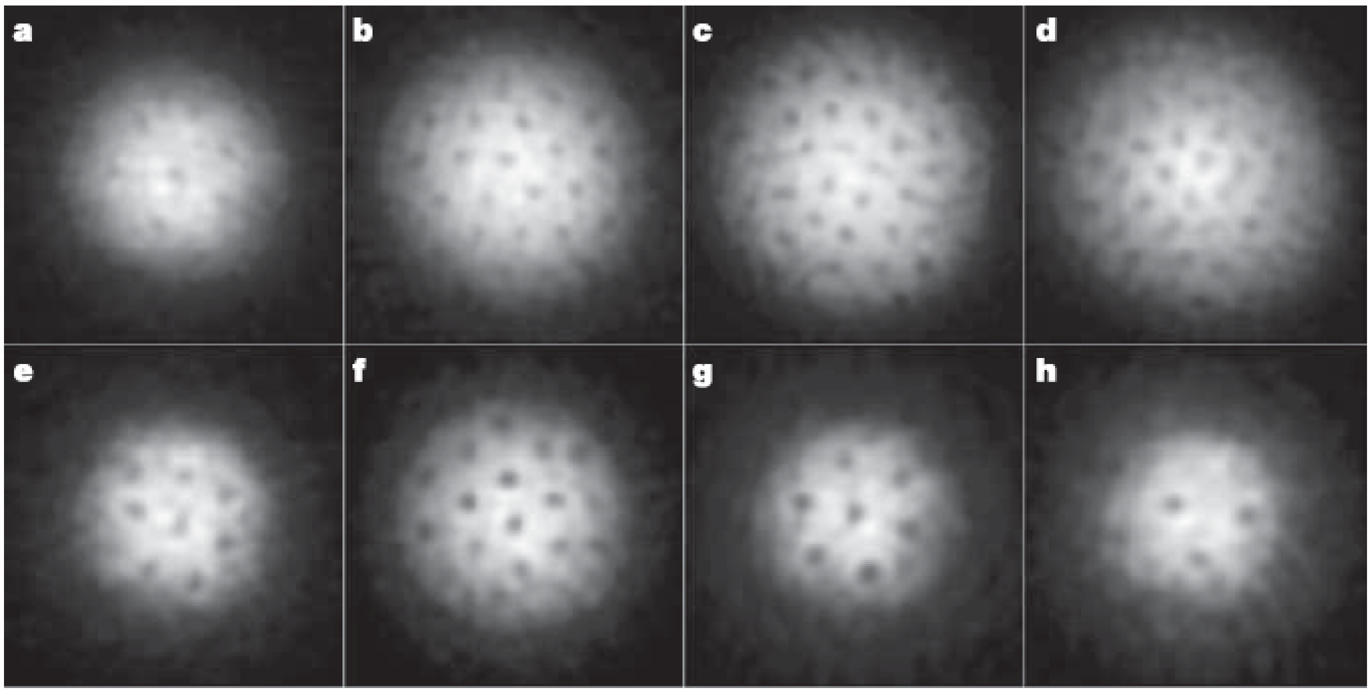
[M. W. Zwierlein, et al, Nature 435, 1047 (2005)]

^{6}Li: $T/T_F > 0.1{\sim}0.2$

"definitive evidence for superfluidity"

BEC side ←

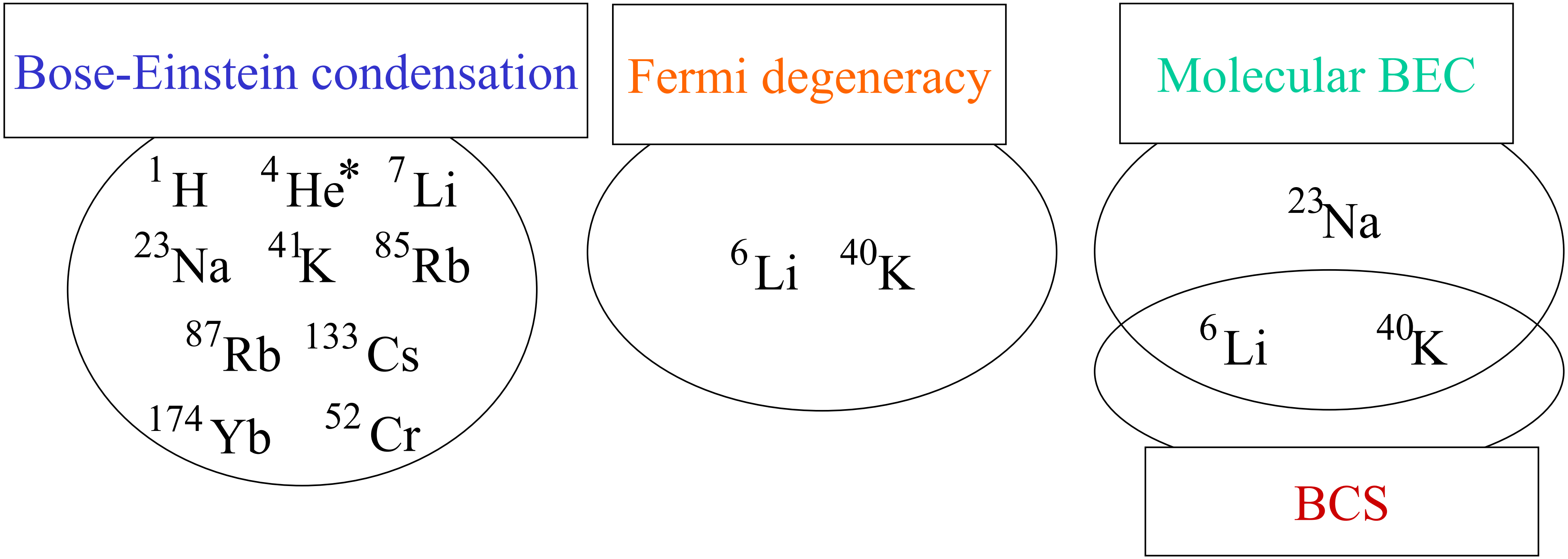
B=745G 766G 792G 812G



833G 843G 853G 863G

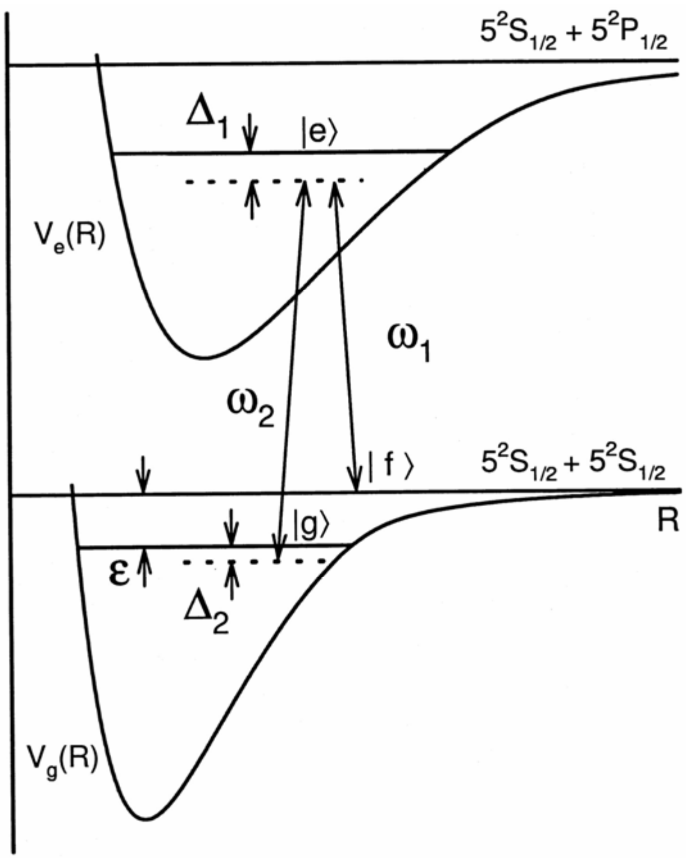
→ BCS side

Atomic Quantum Degenerate Gases

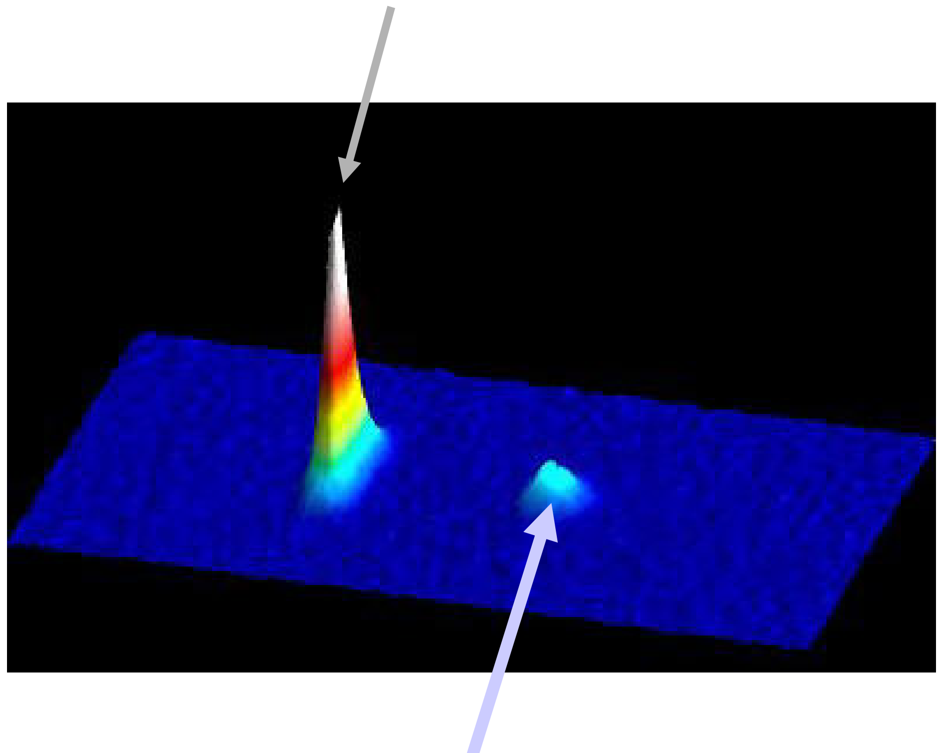


分子のBEC

Rb分子の光会合



Cs 原子BEC



Cs 分子"BEC"