Interlayer coherence in $\nu=1$ and $\nu=2$ bilayer quantum Hall states

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The presence of interlayer coherence in bilayer quantum Hall states was examined by magnetotransport experiments. Two macroscopic quantum conjugate observables, the phase difference and the electron density difference between the two layers, are experimentally addressed by tilting the sample in a magnetic field and applying gate bias voltages. Results strongly indicate the presence of interlayer coherence at the filling factor $\nu=1$ and 2. [S0163-1829(99)05724-0]

Quantum coherence is one of the most important concepts in physics. The macroscopic coherent state is a quantum system closest to the classical one, where the particle number and its conjugate phase are measurable simultaneously with extreme accuracy. Well-established examples are superconductors and superfluids. The quantum Hall (QH) state is a new candidate because of its similarity to the superconductor.[1] Although quantum coherence does not develop in the monolayer QH system because it is an incompressible fluid, there exists an intriguing possibility[2,3] that bilayer quantum Hall (BLQH) states exhibit coherence. This is because of an additional degree of freedom, i.e., the number difference and the conjugate phase difference between the two layers.

In order to experimentally examine the presence of interlayer coherence, it is necessary to address simultaneously the two macroscopic conjugate observables, θ and σ, where θ is the interlayer phase difference and σ is the normalized density difference σ defined by $\sigma=(n_f-n_b)/n_t$ with n_t the total number density, and n_f and n_b the electron density in the front and back layers. The essential property is that they are determined simultaneously in the macroscopic system, since their uncertainties are negligible, $\Delta\theta\propto 1/\sqrt{n_t}\rightarrow 0$ and $\Delta\sigma \propto 1/\sqrt{n_t}\rightarrow 0$.

The interlayer phase difference θ, if it exists, can be tuned by applying a parallel magnetic field between the two layers, which can be achieved by tilting the bilayer system. The interlayer number difference σ can be controlled by applying gate bias voltages to the two layers. When the interlayer coherence exists, the BLQH state persists even if the electron density is arbitrarily unbalanced between two quantum wells.

Murphy *et al.*[4] have found an activation-energy anomaly together with a phase transition in the $\nu=1$ BLQH state by increasing the parallel magnetic field. Sawada *et al.*[5] have found the anomalous stability of the state by applying bias voltages at $\nu=1$ and 2. Although these two experiments suggest the presence of the coherence,[6–8] one needs to make a simultaneous determination of both of the conjugate observables to elucidate the coherent nature of the BLQH system.

In this paper, we report the results of experiments on the $\nu=1$ and 2 BLQH states, where we have measured the Hall-plateau width and the activation energy by changing the density in each quantum well and simultaneously tilting the sample in a magnetic field. Our results strongly indicate the presence of interlayer coherence at $\nu=1$ and 2.

The sample was grown by molecular beam epitaxy on a (100)-oriented GaAs substrate, and consists of two modulation doped GaAs quantum wells of width $W=200$ Å, separated by an $Al_{0.3}Ga_{0.7}As$ barrier of thickness $d_B=31$ Å. The total electron density of this sample was 2.3×10^{11} cm^{-2} at zero gate voltage, the mobility was 3.0×10^{5} cm^2/V s at temperature $T=30$ mK, and the tunneling-energy gap Δ_{SAS} was 6.8 K. The Schottky gate electrodes were fabricated on both front and back surfaces of the sample so that the front-layer and the back-layer electron density can be independently controlled by adjusting the front and the back gate voltage.

Measurements were performed with the sample mounted in a mixing chamber of a dilution refrigerator. The magnetic field with maximum 13.5 T was applied to the sample. Standard low-frequency ac lock-in techniques were used with

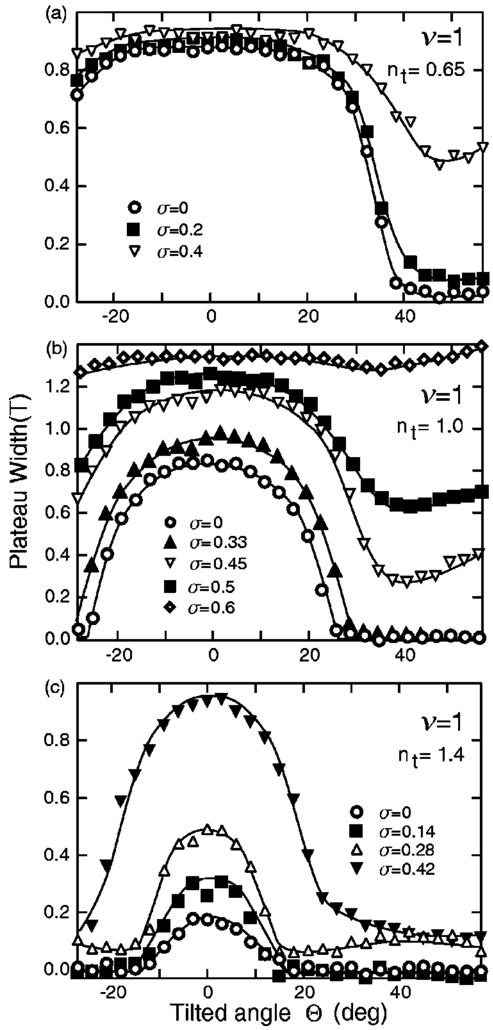

FIG. 1. The Hall-plateau width of the $\nu=1$ state at 50 mK as a function of the tilted angle Θ at various n_t and σ.

currents less than 100 nA to avoid heating effects. The sample mounted on a goniometer with the superconducting stepper motor[9] can be rotated into any direction in the magnetic field.

The Hall-plateau width has previously been shown to be a good indicator of the stability of the QH state, and a close correlation with the activation energy has been pointed out.[5] Its dependence on the tilted angle Θ and the normalized density difference σ gives an overview in categorizing different types of BLQH states.

In Fig. 1 we show the plateau width of the $\nu=1$ BLQH state as a function of Θ at various n_t and σ. The plateau width is defined[10] with respect to the perpendicular field $B_\perp$. All data of the plateau width exhibit a similar behavior.

We give the activation energy as a function of Θ in Fig. 2. As typical examples we show the data with $n_t=1.0$ and 0.7 in units of 10^{11} cm^{-2}. Two curves are at the balanced point ($\sigma=0$) and one at the unbalanced point ($\sigma=0.45$). The activation energy Δ is derived from the temperature dependence of the magnetoresistance; $R_{xx}=R_0\exp(-\Delta/2T)$. (This definition is different by a factor of 2 from the previous one.[5])

The activation energy has a peak at $\Theta=0$, and drops rapidly to a certain tilted angle Θ^*, and then it becomes flat

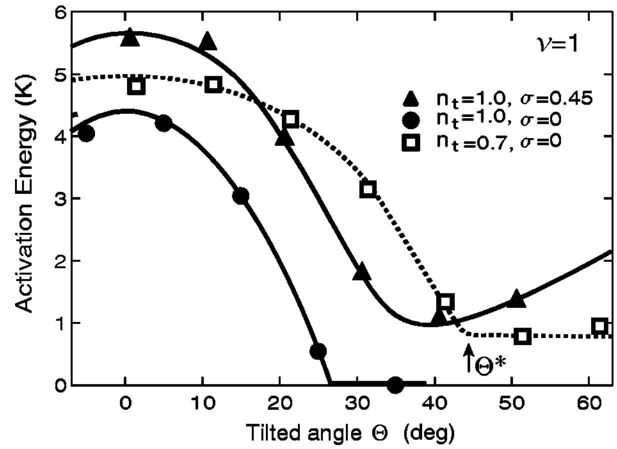

FIG. 2. Activation energy of the $\nu=1$ state as a function of Θ at various n_t and σ. The total density n_t is in units of 10^{11} cm^{-2}.

($\sigma=0$) or increases ($\sigma\neq 0$). This behavior is the anomaly revealed first by Murphy *et al.*[4] at the balanced point ($\sigma=0$). The critical angle Θ^* clearly indicates a phase transition. Yang *et al.*[7] have argued that it is the commensurate state for $\Theta<\Theta^*$ and the incommensurate state for $\Theta>\Theta^*$,

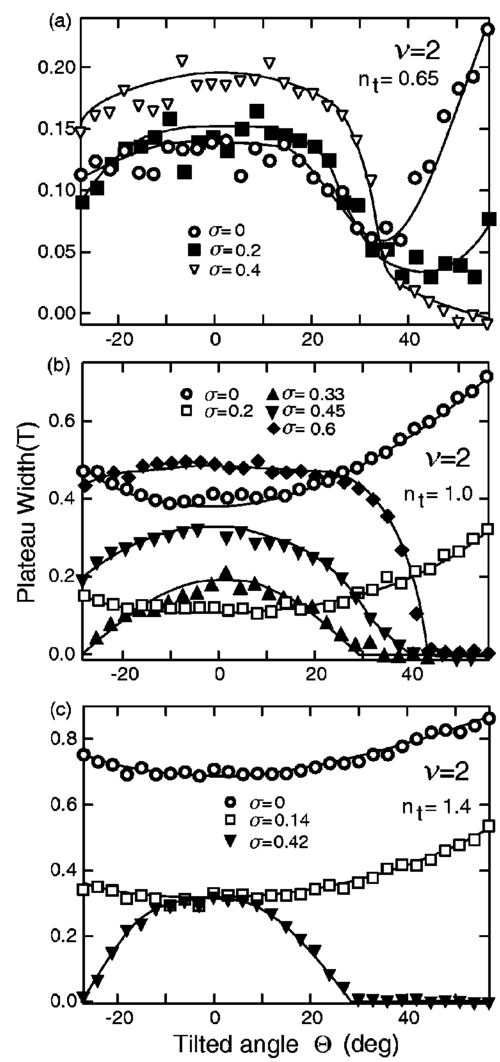

FIG. 3. The Hall-plateau width of the $\nu=2$ state at 50 mK as a function of Θ at various n_t and σ.

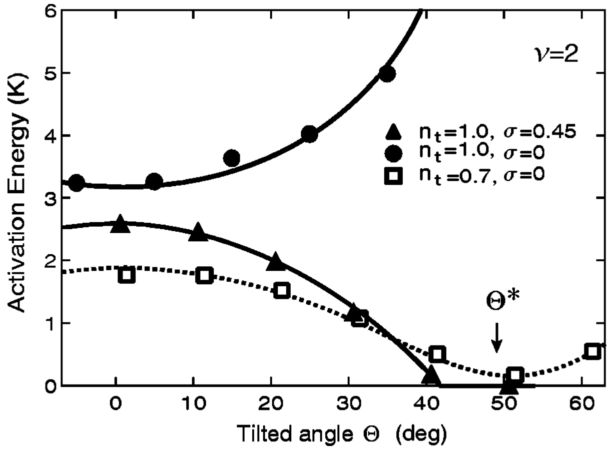

FIG. 4. Activation energy of the $\nu=2$ QH state as a function of Θ at various n_t and σ.

about which we explain later based on Eq. (1). We also identify Θ^* with the commensurate-incommensurate (CIC) phase transition point. Our new finding is that the CIC transition occurs also in unbalanced configurations ($\sigma\neq 0$). As we argue later in Eq. (2), the phase difference θ is related to Θ in the interlayer coherent phase. Thus, each BLQH state turns out to possess definite values of σ and θ in Fig. 1: Namely, their uncertainties are negligible, $\Delta\theta\to 0$ and $\Delta\sigma\to 0$. We conclude that this is evidence of the development of the interlayer coherence at $\nu=1$.

We next show the plateau width of the $\nu=2$ BLQH state in Fig. 3. There are two distinct behaviors, as is consistent with the previous data.[5] (A) The overall behavior at a low density [Fig. 3(a)] bears a close resemblance to that in the $\nu=1$ state (Fig. 1). It indicates that the interlayer coherence has developed also at $\nu=2$ together with a possible CIC transition. (B) At higher densities [Figs. 3(b) and 3(c)], we observe two distinct types of states: (B1) The plateau width near the balanced point ($\sigma\simeq 0$) increases monotonously as the tilted angle increases; (B2) the plateau width at large off-balanced points shows a behavior characteristic to the coherent state.

We give the activation energy as a function of Θ in Fig. 4, where $n_t=1.0$ and 0.7 in units of 10^{11} cm^{-2}. (A) At low density ($n_t=0.7$) it shows an anomalous behavior in the activation energy as in the $\nu=1$ coherent BLQH state. However, the activation energy begins to increase beyond Θ^*, whose origin will be the Zeeman energy of spin excitations as we discuss later (see Table I). (B1) At higher density ($n_t=1.0$) it increases monotonically at the balanced point ($\sigma=0$). This is an expected behavior in the compound state which is stable only around the balanced point.[5] The increase is due to the Zeeman energy of spin excitations. Note that no tunneling energy contributes to the compound state. (B2) At the off-balanced point ($\sigma=0.45$) its behavior is that of a typical coherent state established in the $\nu=1$ BLQH state.

We proceed to discuss physics behind the interlayer coherence of the bilayer QH states. The interlayer coherence is described by the Hamiltonian density,[7,11]

$$\mathcal{H}=\frac{\rho_s}{2}[(\partial_x\theta)^2+(\partial_x\sigma)^2]+\frac{e^2n_t^2}{8C}\sigma^2 -\frac{\Delta_{SAS}n_t}{4}\sqrt{1-\sigma^2}\cos(\theta-Qx)-\frac{en_tV_{\text{bias}}}{2}\sigma, \qquad (1)$$

TABLE I. Comparison between the optical and magnetotransport results at $\nu=2$.

	our work	Pellegrini *et al.*
Sample	$\Delta_{SAS}=6.8$ K $n_t=0.6$–1.6	$\Delta_{SAS}\simeq 7$ K $n_t=0.6$–1.4
Low density ↓ High density	coherent $\downarrow n_t\simeq 0.9$ compound	unpolarized $\downarrow n_t\simeq 1.3$ polarized
Low tilt ↓ High tilt	commens. $\downarrow\Theta^*\simeq 50°$ incommens. or compound	unpolarized $\downarrow\Theta^*\simeq 37°$ polarized

where $Q=2\pi dB_\parallel/\phi_0$ with the Dirac flux unit $\phi_0\equiv h/e$. We have taken the BLQH system parallel to the xy plane and applied the parallel magnetic field to the y direction. The first term describes the Coulomb exchange energy with the pseudospin stiffness $\rho_s\simeq\nu e^2/(16\sqrt{2\pi}\varepsilon l_B)$; the second term the capacitive charging energy with the capacitance C; the third term the tunneling energy; the last term describes the bias voltage applied externally. The tilted angle Θ is given by $\tan\Theta=B_\parallel/B_\perp$. The phase difference θ induces screening currents $J_x^f=-J_x^b=(2\pi e\rho_s/h)\partial_x\theta$ on the two layers into the opposite directions.[6]

On one hand, in the commensurate phase ($\Theta<\Theta^*$) the tunneling term is minimized, as $\theta=Qx$ yields, or

$$\theta(x)=2\pi xdB_\parallel/\phi_0=2\pi xdB_\perp\tan\Theta/\phi_0. \qquad (2)$$

The phase difference $\theta(x)$ counts the number of flux penetrated into the area xd of the junction. As the tilted angle increases, the screening currents $|J_x^{f,b}|$ increase, and they will decrease the activation energy by destabilizing excitations[8] across the two layers. On the other hand, in the incommensurate phase ($\Theta>\Theta^*$) the kinetic term is minimized, and yields $\theta=\theta_0=$const. No screening current flows, which means that the activation energy is insensitive to the tilted angle. The critical angle Θ^* is given at the balanced point[7] by $\tan\Theta^*=(1/2\pi^2d)\sqrt{\Delta_{SAS}/n_t\rho_s}$. It decreases as n_t increases, as is qualitatively consistent with the data (Fig. 1).

An important observation is that $\cos(\theta_0-Qx)$ oscillates very rapidly in the incommensurate phase, and its average vanishes. Consequently, the tunneling energy is suppressed as a many-body effect, and the second energy level is given by the antisymmetric spin-up state at the balanced point [Fig. 5(b)]. Because charge excitations do not acquire the Zeeman energy, the activation energy is flat in the incommensurate

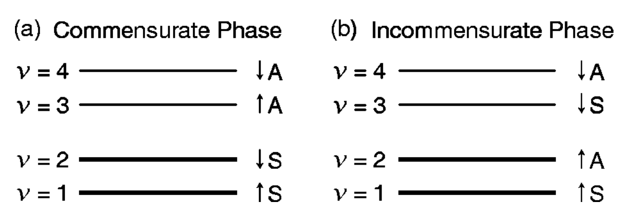

FIG. 5. The alignment of energy levels of electrons at $\nu=2$. The symmetric and antisymmetric states are represented by S and A. The short vertical arrows represent the orientations of electron spin.

phase at $\nu=1$ in the balanced configuration, as the data (for $\sigma=0$) in Fig. 2 and also the data by Murphy *et al.*.[4] explain.

We now discuss the BLQH state at $\nu=2$. When the total density is sufficiently small, the interlayer coherence develops[8] as in the $\nu=1$ state, and is described also by the effective Hamiltonian (1). Because the tunneling energy gap Δ_{SAS} is larger than the Zeeman energy $(g^* \mu_B B)$ in our sample ($\Delta_{SAS}/g^* \mu_B B \simeq 4$ at $B=5$ T), the lowest two levels occupied are the symmetric spin-up and spin-down states in the commensurate phase [Fig. 5(a)]. The activation energy will decrease as the tilted angle increases as in the $\nu=1$ commensurate phase. There are two possible phases for a large tilted angle. First, the transition may be from the coherent commensurate phase to the coherent incommensurate phase. In the incommensurate phase, since the tunneling interaction is effectively suppressed by a many-body effect, the lowest two levels are the symmetric and antisymmetric spin-up states [Fig. 5(b)]. Second, the transition may be from the coherent commensurate phase to the compound phase. Such a transition is also possible as a result of the decrease of the one-body tunneling interaction. In any of them, charge excitations flip spins, as is seen in the increase of the activation energy (for $n_t=0.7$ and $\sigma=0$) in Fig. 4.

Any of the physical interpretations of our magnetotransport experiments is supported by the results of optical experiments.[12] We summarize the relations between the results in these two types of experiments in Table I. We also remark that our experimental results are consistent with theoretical results at $\nu=2$ due to Das Sarma, Sachdev, and Zheng,[13] since the interlayer coherent state and the compound state correspond to the canted state and the FM state.

In conclusion, by tilting the sample in a magnetic field and applying gate bias voltages, we have experimentally approached the two quantum conjugate observables in searching for interlayer coherence in BLQH systems. All our experimental data are consistent with the presence of coherence. Our data at $\nu=2$ suggest that either the incommensurate phase or the compound phase is realized for a large tilted angle. A further experiment is needed to clarify this point.

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